TECHNICAL NOTE R-61

COMPUTATION OF PLUG NOZZLE CONTOURS BY THE RAO OPTIMUM THRUST METHOD

July 1963 (NASA CR-21914; R-61)

Prepared For

ADVANCED PROPULSION SECTION
PROPULSION AND MECHANICS BRANCH
P & VE DIVISION
GEORGE C. MARSHALL SPACE FLIGHT CENTER

By

SCIENTIFIC RESEARCH LABORATORIES 1428002
BROWN ENGINEERING COMPANY, INC., Huntsville, Ala

(NASA Contract No. NAS8-5289)

Prepared By

C. C. Lee July 1963 75 p 8 refs

ABSTRACT

21914

The purpose of this study was to develop a FORTRAN computer program to design a plug nozzle by using Rao's maximum thrust theory.

The control surface of the plug nozzle is determined by Lagrange multipliers to obtain optimum thrust. The flow field properties are calculated by using the method of characteristics. A streamline that passes through the end point of the control surface forms the plug contour.

Approved by:

C. E. Kaylor

C. E. Kaylor
Director
Gas Dynamics/
Thermochemistry Laboratory

LIST OF SYMBOLS

A^*	area of nozzle throat
C_F	vacuum thrust coefficient
c^*	aerodynamic reference speed at sonic velocity
M	Mach number
M^*	dimensionless speed, $\frac{w}{c^*}$
p	pressure
R, x	coordinates
T	thrust
w	velocity magnitude

<u>Greek Symbols</u>

α	Mach angle
β	compatibility coefficient
γ	ratio of specific heats
ε	expansion ratio
η	dimensionless compatibility coefficient
θ	flow inclination angle
λ	dimensionless geometric coefficient
ρ	mass density
ϕ	angle between control surface and nozzle axis

<u>Subscripts</u>

a	ambient condition
b	plug base condition
c	combustion chamber condition

D	signifies the end point of the plug
E	signifies the lip of the shroud
L	properties on left running characteristic
R	properties on right running characteristic
T	signifies the throat of the plug nozzle
*	signifies the throat condition

TABLE OF CONTENTS

LIST OF FIGURES

INTRODUCTION

The basic purpose of this work was to develop a FORTRAN program to design a plug nozzle contour for maximum thrust with specified length based on the theory presented in Reference 1. A detailed derivation of the theory and a description of the computer program are given in this report.

The gas is treated as inviscid, and the expansion process is assumed to be isentropic and adiabatic. The base pressure at the end of the plug is assumed to be variable. The flow field very near the lip of the shroud was computed by the Prandtl-Meyer relation; the remainder of the flow field was determined by the method of characteristics.

DERIVATION OF THEORY

Flow Field Calculation

The gas expansion process in the flow field of the plug nozzle is assumed to be isentropic, adiabatic, and frictionless. The method of characteristics is logically and physically applicable for determining pertinent parameters throughout the flow field of a supersonic isentropic plug nozzle.

There are two families of characteristic equations. The left running characteristic equations are

$$\frac{dR}{dx} = \tan\,(\theta + \alpha) \quad , \tag{1}$$

$$d\theta - \cot\alpha \frac{dw}{w} + \frac{\sin\theta \sin\alpha}{\sin\,(\theta + \alpha)} \frac{dR}{R} = 0 \quad . \tag{2}$$

The right running characteristic equations are

$$\frac{dR}{dx} = \tan\,(\theta - \alpha) \quad , \tag{3}$$

$$d\theta + \cot\alpha \frac{dw}{w} - \frac{\sin\theta \sin\alpha}{\sin\,(\theta - \alpha)} \frac{dR}{R} = 0 \quad . \tag{4}$$

In a plug nozzle, the flow angle θ is always negative, the Mach angle α is always positive. Therefore the value of $(\theta + \alpha)$ in certain regions might be equal to or close to zero. In this event, Equation (2) would present a problem in iteration. To eliminate this problem,

Equation (1) can be written as

$$\frac{dR}{\sin(\theta + \alpha)} = \frac{dx}{\cos(\theta + \alpha)} \quad . \tag{5}$$

The following relation is also useful:

$$\frac{dw}{w} = \frac{dM^*}{M^*} \quad . \tag{6}$$

Now Equations (2) and (4) can be rewritten as

$$d\theta - \cot\alpha \frac{dM^*}{M^*} + \frac{\sin\theta \sin\alpha}{\cos(\theta + \alpha)} \frac{dx}{R} = 0 \quad , \tag{7}$$

$$d\theta + \cot\alpha \frac{dM^*}{M^*} - \frac{\sin\theta \sin\alpha}{\sin(\theta - \alpha)} \frac{dR}{R} = 0 \quad . \tag{8}$$

Subscribe L and R on the parameters for the left and right running characteristic equations, and N is used as subscript on the parameters in solution. By combining Equations (1), (3), (7), and (8), x, R, M*, θ can be solved numerically. Let

$$\lambda_L = \tan(\theta_L + \alpha_L) \quad , \tag{9}$$

$$\eta_L = \frac{\cot(\alpha_L)}{M^*_L} \quad , \tag{10}$$

$$\beta_L = \frac{\sin(\theta_L)\sin(\alpha_L)}{R_L \cos(\theta_L + \alpha_L)} \quad , \tag{11}$$

$$\lambda_R = \tan(\theta_R - \alpha_R) \quad , \tag{12}$$

$$\eta_R = \frac{\cot(\alpha_R)}{M^*_R} \quad , \tag{13}$$

$$\beta_R = \frac{\sin(\theta_R)\sin(\alpha_R)}{R_R \sin(\theta_R - \alpha_R)} \quad . \tag{14}$$

The approximate solutions can be written as follows:

$$x_N = \frac{(\lambda_R x_R - \lambda_L x_L) + (R_L - R_R)}{\lambda_R - \lambda_L} \quad , \tag{15}$$

$$R_N = R_L - \lambda_L (x_L - x_N) \quad , \tag{16}$$

$$M^*_N = \frac{\theta_R - \theta_L + \eta_L M^*_L + \eta_R M^*_R - \beta_R (R_R - R_N) - \beta_L (x_L - x_N)}{\eta_L + \eta_R} \quad , \tag{17}$$

$$\theta_N = \theta_L - \eta_L (M^*_L - M^*_N) + \beta_L (x_L - x_N) \quad . \tag{18}$$

If the parameters on the control surface are known, the characteristic net, as shown in Figure 1, can be constructed, and the parameters can be determined throughout the flow field. The coefficients, λ, η, β, can be averaged between points L and N, and between R and N to carry on the iterations until the tolerance of θ_N is within desired limit.

<u>Boundary Conditions</u>

In order to obtain maximum thrust for a plug nozzle with a prescribed length, Lagrange multipliers were used to determine the

extreme values of the thrust function. The constraining relations are that the axial distance of the control surface (Figure 2) is held constant, and the mass flow crossing the control surface is equal to mass flow through the throat, namely,

$$\int_D^E \cot(-\phi)\, dR = \text{constant} \quad , \tag{19}$$

$$\int_D^E \rho\, [w \sin(-\phi + \theta)] \left[\frac{2\pi R\, dR}{\sin(-\phi)}\right] = \text{constant} \; . \tag{20}$$

The thrust of the plug nozzle is given by

$$\begin{aligned} T = \int_D^E p\, 2\pi R\, dR + \int_D^E \rho \left[w \sin(-\phi + \theta)\, \frac{2\pi R\, dR}{\sin(-\phi)}\right] w \cos(-\theta) \\ - p_a\, \pi R_E^2 + p_b\, \pi R_D^2 \quad . \end{aligned} \tag{21}$$

Multiply Equation (19) by λ, and Equation (20) by λ_2, add the results to Equation (21), and one obtains

$$\begin{aligned} I = \int_D^E \Bigg\{ \left[p + \rho w^2\, \frac{\sin(-\phi + \theta)\cos(-\theta)}{\sin(-\phi)}\right] 2\pi R \,+ \\ \lambda_2 \left[\rho w\, \frac{\sin(-\phi + \theta)}{\sin(-\phi)}\right] 2\pi R + \lambda_3 \cot(-\phi) \Bigg\}\, dR \\ - \pi R_E^2\, p_a + \pi R_D^2\, p_b \quad . \end{aligned} \tag{22}$$

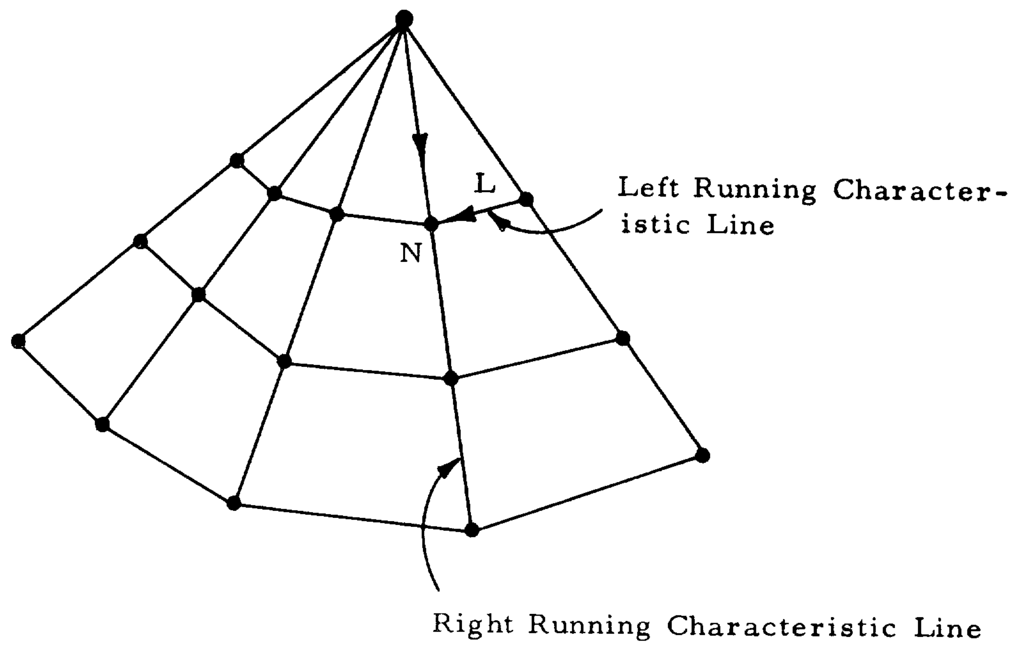


Figure 1 - Characteristic Net

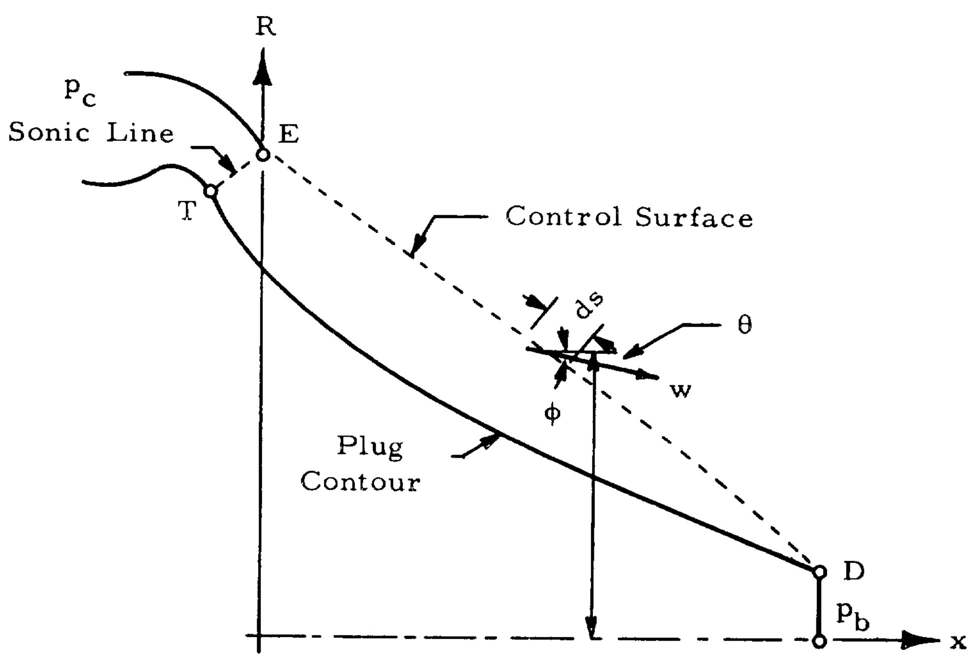


Figure 2 - Sketch of a Plug Nozzle

Let

$$F = \left[p + \rho w^2 \frac{\sin(-\phi + \theta)\cos(-\theta)}{\sin(-\phi)}\right] 2\pi R + \lambda_2 \left[\rho w \frac{\sin(-\phi + \theta)}{\sin(-\phi)}\right] 2\pi R$$

$$+ \lambda_3 \cot(-\phi) \tag{23}$$

From Euler's Equation

$$\frac{\partial F}{\partial \theta} = \frac{\rho w^2 \pi R}{\sin(-\phi)} \left[\cos(-\theta)\cos(-\phi + \theta) + \sin(-\phi + \theta)\sin(-\theta)\right]$$

$$+ \frac{\lambda_2 \rho w \pi R}{\sin(-\phi)} \cos(-\phi + \theta) = 0 \tag{24}$$

or

$$w \cos(-\theta - a) = -\lambda_2 \cos a \quad , \tag{25}$$

$$-\lambda_2 = w \frac{\cos(-\theta - a)}{\cos a} \quad , \tag{26}$$

$$\frac{\partial F}{\partial \phi} = \rho w^2 \frac{-\sin(-\phi)\cos(-\phi + \theta)\cos(-\theta) + \sin(-\phi + \theta)\cos(-\theta)\cos(-\phi)}{\sin^2(-\phi)} 2\pi R$$

$$+ \lambda_2 \rho w 2\pi R \left[\frac{-\sin(\phi)\cos(-\phi + \theta) + \sin(-\phi + \theta)\cos(-\phi)}{\sin^2(-\phi)}\right] \tag{27}$$

$$+ \lambda_3 \csc^2(-\phi) = 0$$

or

$$-\lambda_3 = 2\pi R \rho w^2 \sin\theta \left[\cos(-\theta) - \frac{\cos(-\theta - a)}{\cos a}\right]$$

$$= 2\pi R \rho w^2 \sin^2\theta \tan a \quad , \tag{28}$$

or

$$-\lambda_3 = R \rho w^2 \sin^2 \theta \tan \alpha \quad , \tag{29}$$

$$\frac{\partial F}{\partial w} = 2 w \rho \frac{\sin(-\phi + \theta) \cos(-\theta)}{\sin(-\phi)} 2 \pi R + \lambda_2 \rho \frac{\sin(-\phi + \theta)}{\sin(-\phi)} 2 \pi R = 0 \quad .$$

Substituting λ_2 value into the above equation, one obtains

$$\cos(\theta - \alpha) = 0$$

or

$$\phi = \theta - \alpha$$

and therefore the control surface is a right running characteristic line.

Equations (26) and (29) are two governing equations for the control surface which is the last right running characteristic line. Thus, the flow properties can be obtained along the control surface simply by combining Equations (26) and (27) with

$$\frac{dR}{dx} = \tan(\theta - \alpha) \tag{30}$$

since

$$M = \frac{\frac{2}{\gamma + 1} M^{*2}}{1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}} \tag{31}$$

$$\tan \alpha = \sqrt{\frac{1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}}{M^{*2} - 1}} \quad . \tag{32}$$

Equations (26) and (27) can be rewritten in terms of Mach number and flow angle. From Equation (26)

$$M^* \left[\cos \theta + \sin (-\theta) \tan \alpha\right] = M^*_E \left[\cos \theta_E + \tan \alpha_E \sin (-\theta_E)\right] \tag{33}$$

or

$$M^* \left[\cos \theta + \sin (-\theta) \sqrt{\frac{1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}}{M^{*2} - 1}}\right] = M^*_E \left[\cos \theta_E + \tan \alpha_E \sin (-\theta_E)\right] . \tag{34}$$

From Equation (29)

$$R \left(\frac{\rho}{\rho_c}\right) \left[M^* \sin^2 \theta\right] \tan \alpha = \left[M^*_E \sin \theta_E\right]^2 \tan \alpha_E \frac{\rho_E}{\rho_c} R_E \tag{35}$$

or

$$R \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{\gamma - 1}} \left[M^* \sin \theta\right]^2 \sqrt{\frac{1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}}{M^{*2} - 1}} =$$

$$R_E \left[M^*_E \sin \theta_E\right]^2 \tan \alpha_E \left(1 + \frac{\gamma - 1}{2} M_E^2\right)^{-\frac{1}{\gamma - 1}} . \tag{36}$$

In order to solve the flow parameters on the control surface, the left hand sides of Equations (34) and (36) can be assumed to be constants, A and B, respectively because they are constant in each plug nozzle.

$$M^* \left[\cos \theta - \sin \theta \sqrt{\frac{1 - \frac{\gamma - 1}{\gamma + 1} M^{*2}}{M^{*2} - 1}}\right] = A \quad , \tag{37}$$

$$R\left[1+\frac{\gamma-1}{2}\left(\frac{\frac{2}{\gamma+1}M*^2}{1-\frac{\gamma-1}{\gamma+1}M*^2}\right)\right]^{-\frac{1}{\gamma-1}}[M*\sin\theta]^2\sqrt{\frac{1-\frac{\gamma-1}{\gamma+1}M*^2}{M*^2-1}}=B \tag{38}$$

since

$$\cos\theta=\sqrt{1-\sin^2\theta}\quad .$$

The flow angle can be solved from Equation (37)

$$\sin\theta=\frac{\frac{-2A}{M*}\sqrt{\frac{1-\frac{\gamma-1}{\gamma+1}M*^2}{M*^2-1}}+\sqrt{\frac{4A}{M*^2}\left(\frac{1-\frac{\gamma-1}{\gamma+1}M*^2}{M*^2-1}\right)-4\left(\frac{\frac{2}{\gamma+1}M*^2}{M*^2-1}\right)\left(\frac{A^2}{M*^2}-1\right)}}{2\left(\frac{\frac{2}{\gamma+1}M*^2}{M*^2-1}\right)}\ . \tag{39}$$

If a value of x is assigned, the value of R can be computed by using Equation (30), and the Mach number and the flow angle can be computed by using Equations (38) and (39).

Determination of Plug Contour and Throat Location

Once the lattice points in the flow field have been determined, the streamlines can be drawn. A streamline that passes through the end point of the control surface forms the plug contour. Since the known condition is at the end point, the streamline has to be drawn upstream.

A point (a) on the diagonal $\overline{12}$, as shown in Figure 3, has to be chosen in such a way that a straight line passing through (a) with a flow angle at (a) would also pass through point (L). This point (a) can be considered as a point on the streamline that passes through point (L).

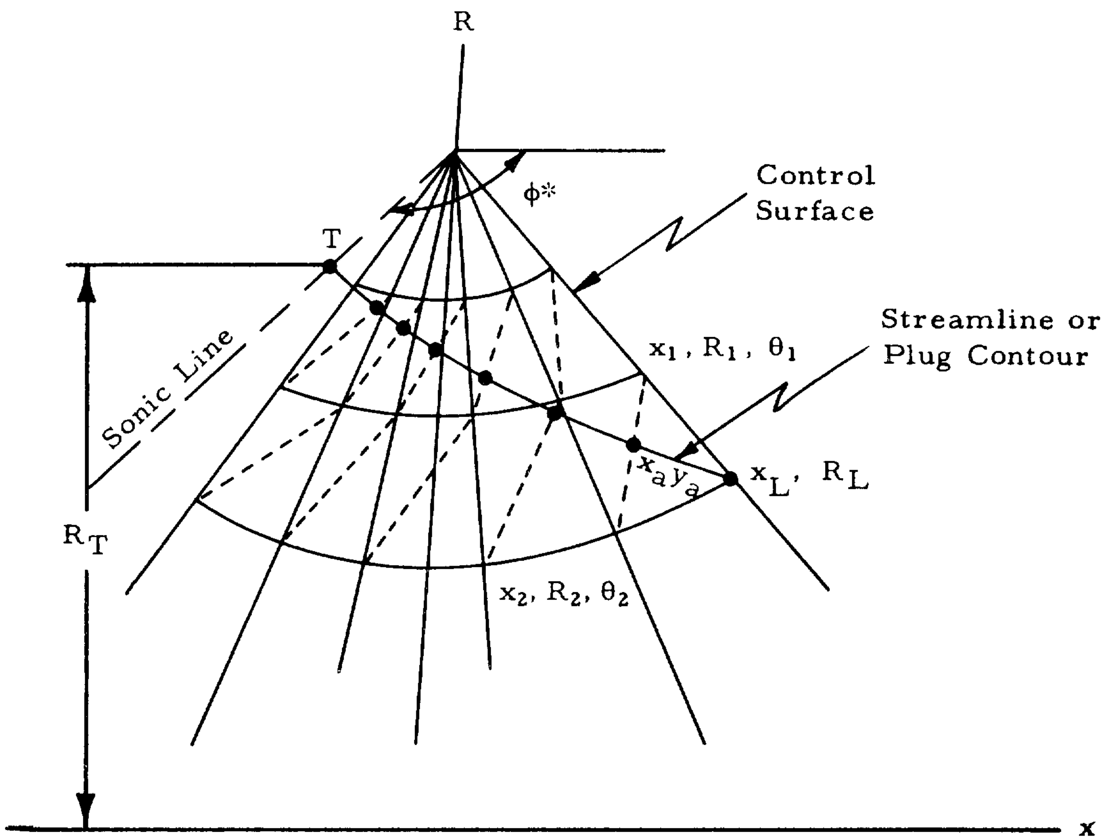


Figure 3 - Illustration of Streamline

Along a streamline

$$\frac{dR}{dx} = \tan\theta \ . \tag{40}$$

A finite difference form can be written for Equation (40)

$$\frac{R_L - R_a}{x_L - x_a} = \tan\theta_a \ . \tag{41}$$

The flow angle at point (a) is obtained by using linear interpolation from the values θ_1 and θ_2

$$\theta_a = m\,\theta_2 + (1 - m)\,\theta_1 \tag{42}$$

where

$$m = \left(\frac{x_1 - x_a}{x_1 - x_2}\right)$$

Thus

$$\theta_a = \left(\frac{x_1 - x_a}{x_1 - x_2}\right)\theta_2 + \left(\frac{x_a - x_2}{x_1 - x_2}\right)\theta_1 \tag{43}$$

since

$$\frac{x_1 - x_a}{R_1 - R_a} = \frac{x_1 - x_2}{R_1 - R_2}$$

or

$$R_a = R_1 - (x_1 - x_a)\left(\frac{R_1 - R_2}{x_1 - x_2}\right) \ . \tag{44}$$

By substituting Equations (43) and (44) into Equation (41), one obtains

$$\frac{R_L - R_1 + (x_1 - x_a)\left(\frac{R_1 - R_2}{x_1 - x_2}\right)}{x_L - x_a} = \tan\left[\frac{x_1 - x_a}{x_1 - x_2}\,\theta_2 + \frac{x_a - x_2}{x_1 - x_2}\,\theta_1\right] . \qquad (45)$$

There is only one unknown, x_a, in Equation (45), and therefore it can be solved numerically. Once x_a is obtained, R_a can be computed from Equation (44).

This procedure is repeated until the streamline has been drawn throughout the flow field.

Since the Mach number M_E and the flow angle θ_E at the lip are known, the flow angle θ^* at the throat can be computed by using the Prandtl-Meyer relation, namely

$$\theta^* = \theta_E - \sqrt{\frac{\gamma+1}{\gamma-1}}\,\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}\,(M_E^2 - 1)} + \tan^{-1}\sqrt{M_E^2 - 1} \ . \qquad (46)$$

The angle of the throat surface can be computed by using the following relation:

$$\phi^* = \theta^* - 90° \ . \qquad (47)$$

The radius R_T of the plug wall at the throat, as shown in Figure 3, can be obtained by using the relation of the conservation of mass

$$\frac{\pi\,(R_E^2 - R_T^2)}{\cos\theta^*} = \frac{\pi\,R_E^2}{\varepsilon} \qquad (48)$$

where

$$\varepsilon = \frac{\pi\,R_E^2}{A^*} \ .$$

Thus

$$R_T = \sqrt{R_E^2 - \frac{R_E^2}{\varepsilon} \cos \theta^*} \; . \tag{49}$$

Governing Conditions

The base pressure at the end of the plug is assumed to be independent of the plug contour. Applying the condition at point D, and substituting λ_2 and λ_3 values into Equation (23), one obtains

$$\frac{(p - p_b)}{\frac{1}{2} \rho w^2} \cot (\alpha) = \sin (-2 \theta) \; . \tag{50}$$

Since

$$w = M \sqrt{\gamma \frac{p}{\rho}} \quad ,$$

$$\cot \alpha = \sqrt{M^2 - 1} \quad ,$$

$$\frac{p}{p_c} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}} \quad ,$$

Equation (50) becomes

$$\frac{2\sqrt{M^2 - 1}}{\gamma M^2} - \left(\frac{p_b}{p_c}\right) \frac{2\sqrt{M^2 - 1}}{\gamma M^2} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} = \sin (-2 \theta) \; . \tag{51}$$

If $p_b = 0$, Equation (51) becomes

$$\frac{2\sqrt{M^2 - 1}}{\gamma M^2} = \sin(-2\theta) \quad . \tag{52}$$

Equation (51) is a necessary condition to determine the end point D. If the Mach number is assigned numerically, Equations (39) and (40) can be used to compute R, θ values along the control surface. Once R and θ values are known, Equation (19) can be used to compute the length of the nozzle,

$$\frac{L}{R_E} = \int_D^E \cot(-\theta + \alpha)\, d\left(\frac{R}{R_E}\right) \quad , \tag{53}$$

and Equation (20) can be used to compute the expansion ratio

$$A^* \rho^* w^* = \int_D^E \rho\,[w \sin(-\phi + \theta)]\left[\frac{2\pi R\, dR}{\sin(-\phi)}\right] \tag{54}$$

or

$$\frac{\pi R_E^{\,2}}{A^*} = \frac{1}{\displaystyle\int_D^E \frac{\rho}{\rho^*}\, \frac{w}{w^*}\, \frac{\sin\alpha}{\sin(-\theta + \alpha)}\, 2\left(\frac{R}{R_E}\right) d\left(\frac{R}{R_E}\right)} \quad . \tag{55}$$

The vacuum thrust coefficient of the plug nozzle can be computed from the following equation:

$$C_F = \frac{T}{p_c A^*} = \frac{\pi R_E^2}{A^*} \int_D^E \left\{\frac{p}{p_c}\left[1 + \frac{\rho w^2}{p}\, \frac{\sin\alpha \cos(-\theta)}{\sin(-\theta + \alpha)}\right]\right\} 2\, \frac{R}{R_E}\, d\left(\frac{R}{R_E}\right) . \tag{56}$$

A series of values of θ_E and M_E in the left hand sides of Equations (35) and (36) is assumed, and various expansion ratios and nozzle lengths would be obtained for each set of assumed values. The expansion ratio $\frac{\pi R_E^2}{A^*}$ vs nozzle length $\frac{L}{R_E}$, with different M_E and θ_E plots is presented in Figures 4, 5, 6A and 6B.

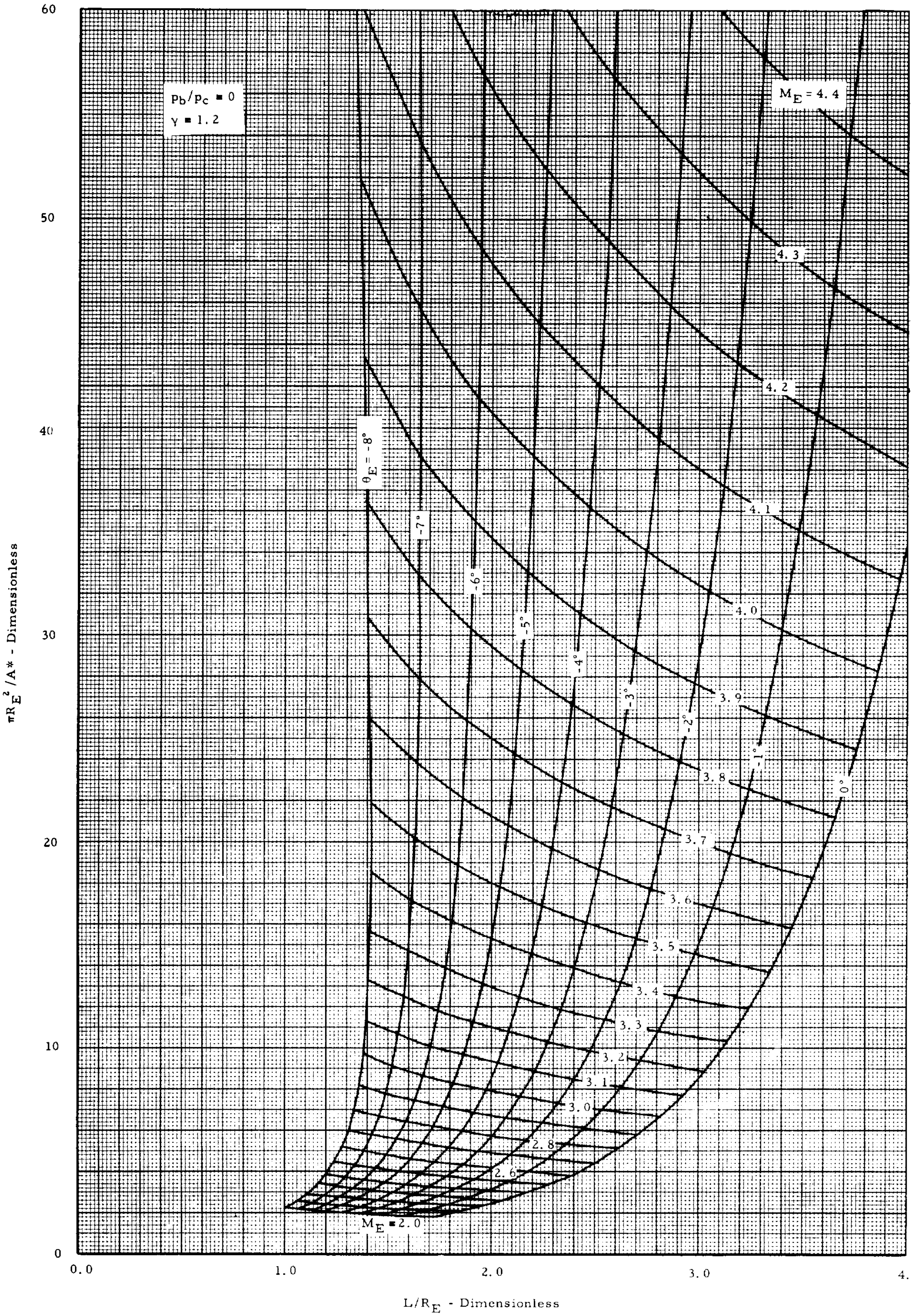


Figure 4

Area Ratio and Length Ratio as Functions of Parameters M_E and θ_E

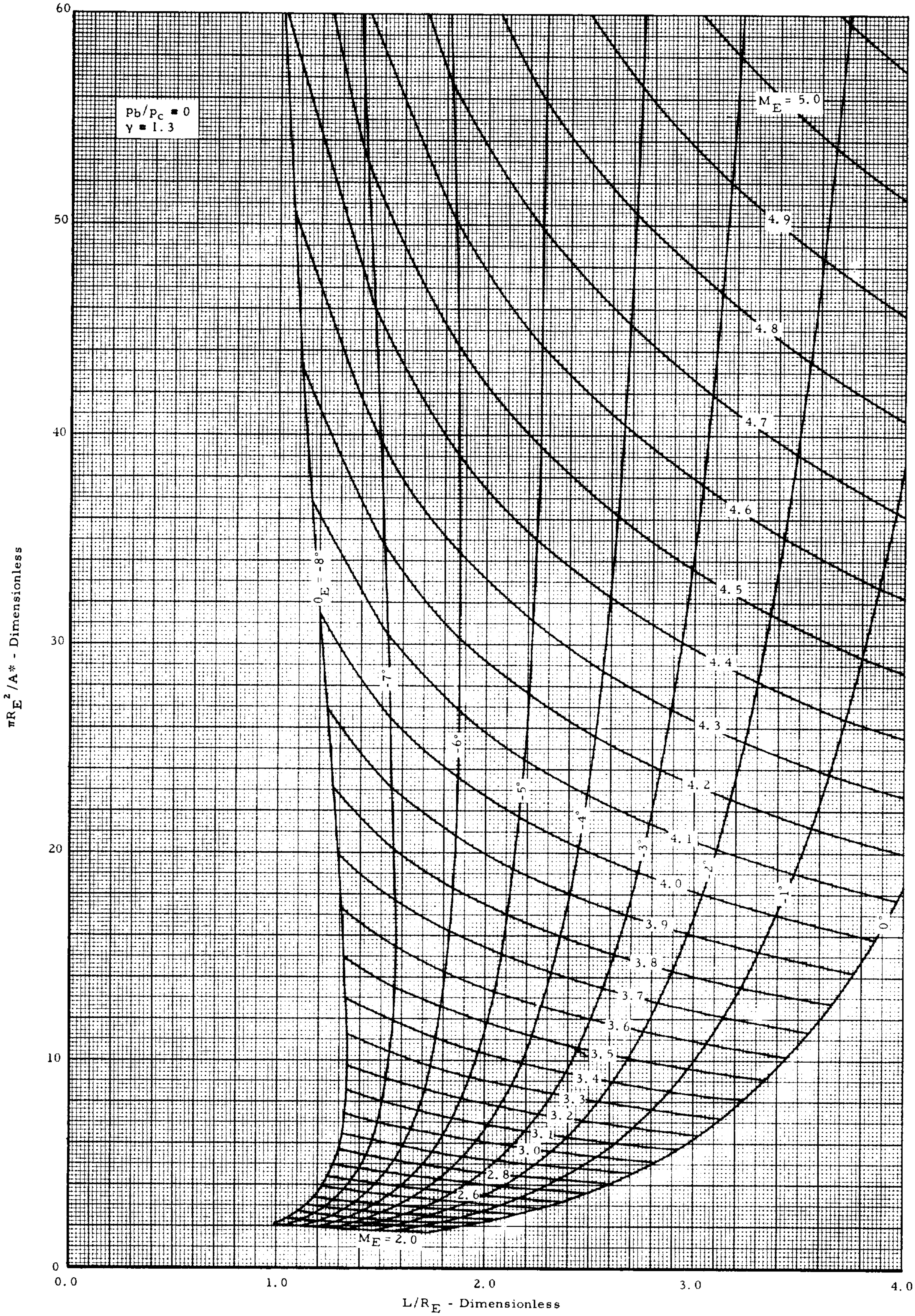


Figure 5

Area Ratio and Length Ratio as Functions of Parameters M_E and θ_E

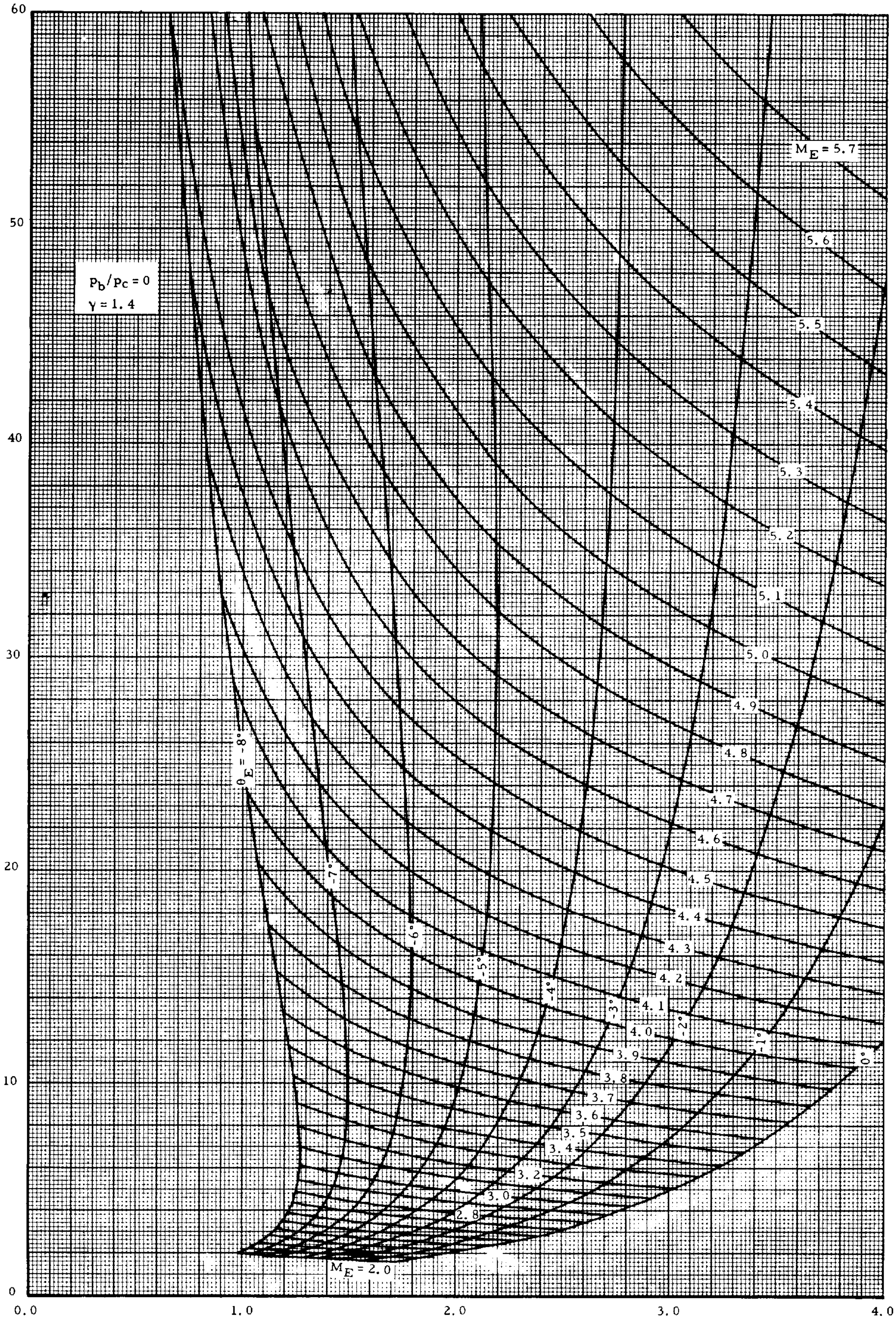


Figure 6A

Area Ratio and Length Ratio as Functions of Parameters M_E and θ_E

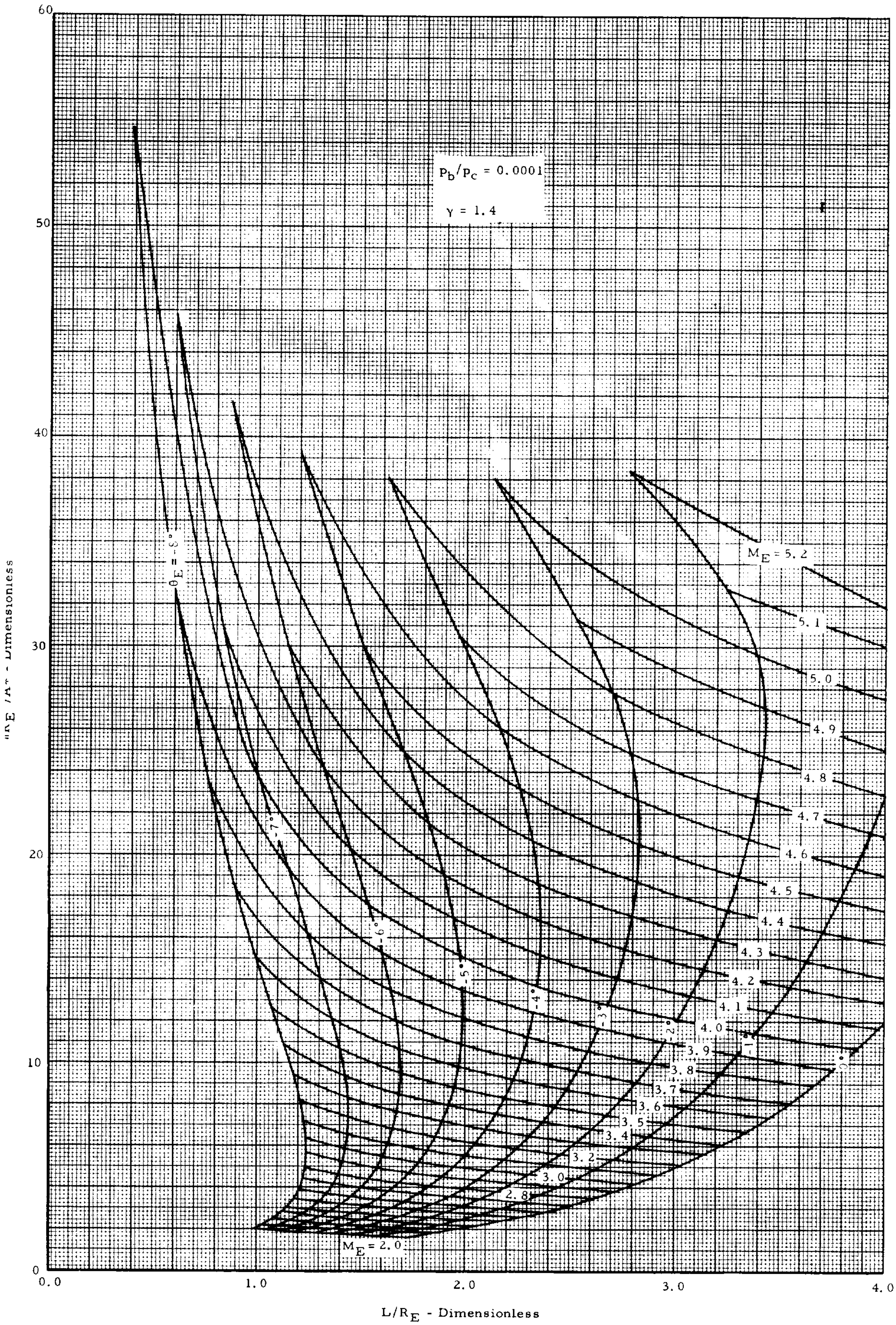


Figure 6B

Area Ratio and Length Ratio as Functions of Parameters M_E and θ_E

DESIGN PROCEDURE

This design method can briefly be divided into four parts:

1. The parameters M_E and θ_E have to be chosen for designing a plug nozzle with a certain expansion ratio, length of the nozzle and the ratio of specific heats. The results that are presented in Figures 4, 5 and 6 serve as a useful tool to select those parameters. If the data are not available in the charts, a FORTRAN program (IBM 7090) which is included in the Appendix can be used to compute the desirable data.

2. Once M_E, θ_E values are known, the flow properties along the control surface can be determined by using the theory stated on page 14.

3. After the flow properties along the control surface are obtained, the method of characteristics as stated on page 2 can be used to determine the flow field. The throat direction is determined by the Prandtl-Meyer relation.

4. A streamline that passes through the end point of the control surface can be determined by using the theory on page 4 . This streamline forms the contour of the plug nozzle. This procedure was shown in Figure 7.

The parts (2), (3) and (4) were also programmed in FORTRAN computer language (IBM 7090), and were arranged in subroutine forms. The programs are included in the Appendix of this report.

Two plug nozzles were designed by using this program, and the shapes of the plug contours were plotted in Figure 8. The coordinates of the contours are presented in Tables I and II.

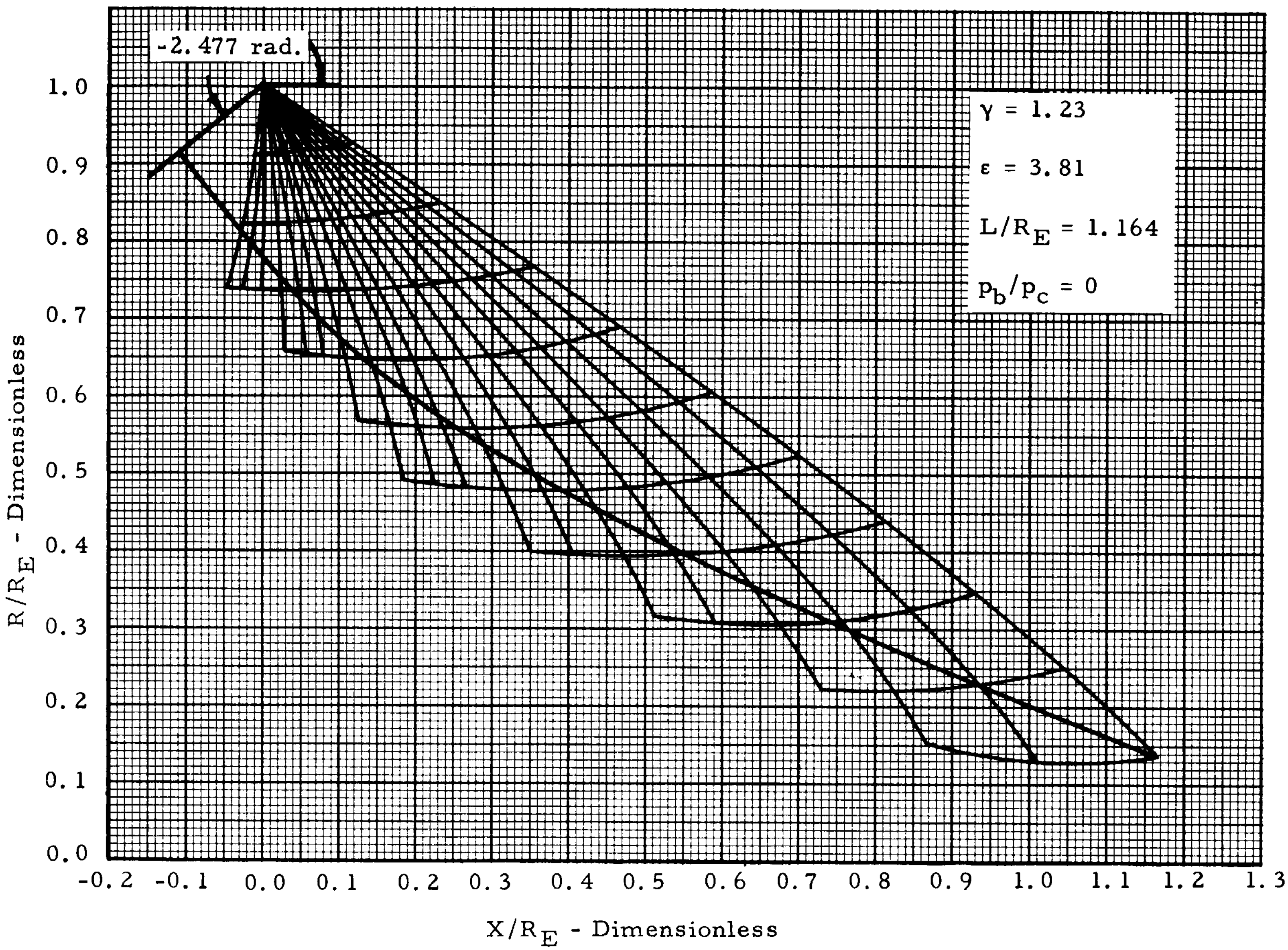


Figure 7

Construction of Plug Nozzle Contour

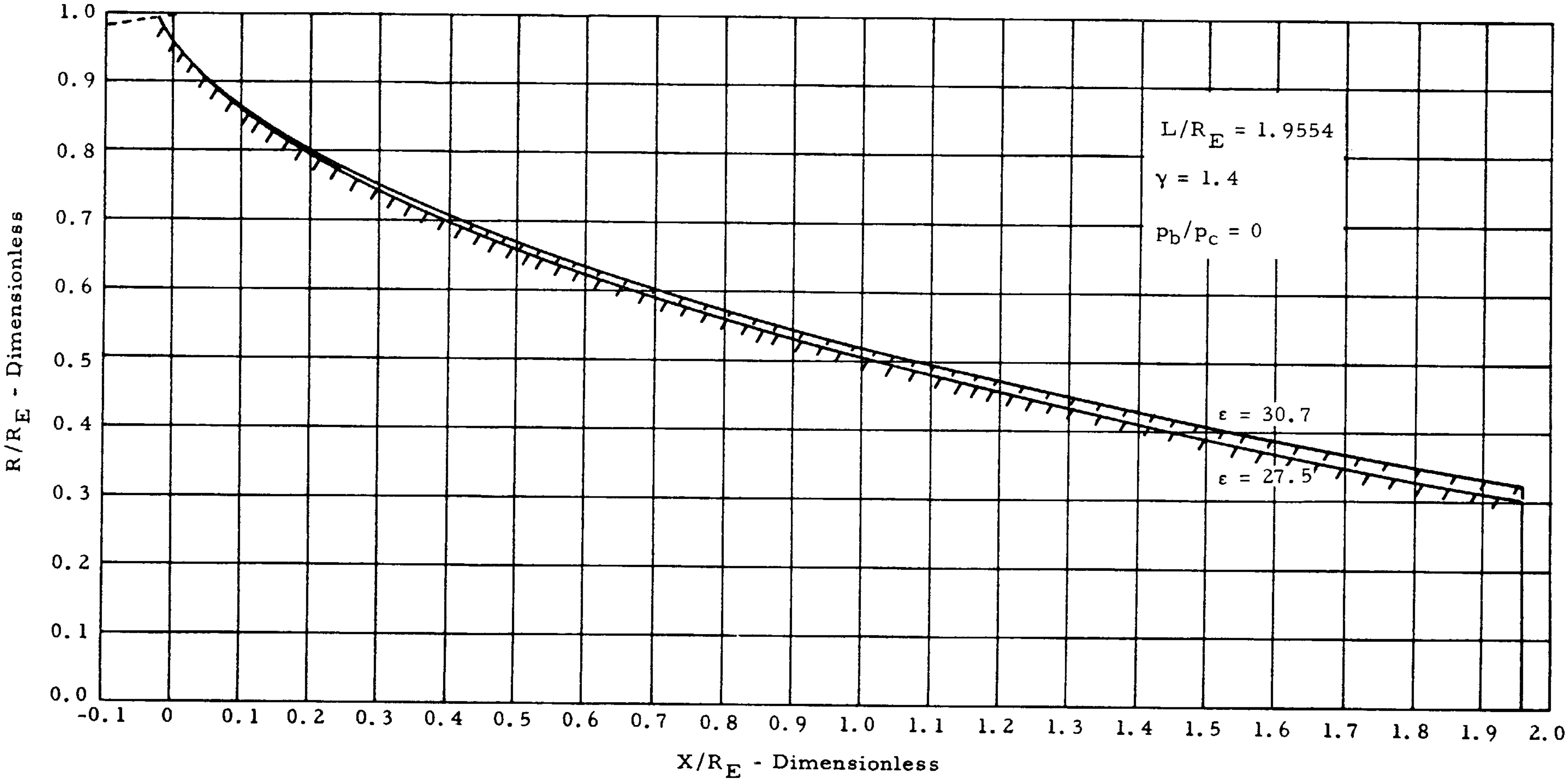


Figure 8

Optimum Thrust Plug Nozzle Contours

TABLE I

PLUG CONTOUR COORDINATES

From Figure 8

$\varepsilon = 30.7$

$L/R_E = 1.9554$

$\gamma = 1.4$

$p_b/p_c = 0$

$M_E = 4.688$

$\theta_E = -5.4$

18 right running characteristic line

20 left running characteristic line

X/R_E	R/R_E	$\tan\theta$
1.9554	0.3238	-0.1540
1.8028	0.3485	-0.1679
1.6744	0.3725	-0.1788
1.5504	0.3958	-0.1883
1.4323	0.4193	-0.1986
1.3168	0.4437	-0.2114
1.2104	0.4674	-0.2227
1.1114	0.4905	-0.2335
1.0139	0.5147	-0.2476
0.9231	0.5384	-0.2612
0.8394	0.5614	-0.2765
0.7582	0.5850	-0.2904
0.6824	0.6082	-0.3066
0.6131	0.6306	-0.3227
0.5456	0.6537	-0.3425
0.4840	0.6760	-0.3622
0.4274	0.6877	-0.3854
0.3735	0.7197	-0.4075
0.3258	0.7403	-0.4321
0.2800	0.7614	-0.4608
0.2397	0.7812	-0.4909
0.2022	0.8009	-0.5253
0.1692	0.8195	-0.5625
0.1391	0.8377	-0.6050
0.1131	0.8546	-0.6521
0.0895	0.8713	-0.7063
0.0703	0.8861	-0.7679
0.0521	0.9013	-0.8392

TABLE I (Cont.)

X/R_E	R/R_E	θ (rad)
0.0380	0.9144	-0.9244
0.0258	0.9269	-1.024
0.0156	0.9386	-1.146
0.0080	0.9484	-1.300
0.0019	0.9576	-1.494
-0.0030	0.9663	-1.746
-0.0064	0.9734	-2.097
-0.00862	0.9791	-2.611

Sonic line

$\theta = -2.953$ rad $\quad R_T/R_E = 0.9969$

TABLE II

PLUG CONTOUR COORDINATES

From Figure 8

$\varepsilon = 27.5$ | $M_E = 4.58$

$L/R_E = 1.9554$ | $\theta_E = -5.5°$

$\gamma = 1.4$ | 18 right running characteristic lines

$p_b/p_c = 0$ | 20 left running characteristic lines

X/R_E	R/R_E	tan θ
1.9554	0.3051	-0.1614
1.8079	0.3310	-0.1751
1.6743	0.3558	-0.1860
1.5508	0.3799	-0.1953
1.4330	0.4041	-0.2054
1.3177	0.4293	-0.2182
1.2116	0.4536	-0.2295
1.1131	0.4773	-0.2400
1.0156	0.5021	-0.2542
0.9251	0.5263	-0.2679
0.8417	0.5497	-0.2809
0.7605	0.5739	-0.2970
0.6849	0.5976	-0.3133
0.6158	0.6203	-0.3294
0.5481	0.6440	-0.3494
0.4868	0.6606	-0.3690
0.4299	0.6888	-0.3906
0.3761	0.7111	-0.4146
0.3286	0.7320	-0.4392
0.2824	0.7536	-0.4683
0.2422	0.7737	-0.4984
0.2044	0.7938	-0.5331
0.1715	0.8126	-0.5704
0.1410	0.8313	-0.6131
0.1150	0.8484	-0.6602
0.0909	0.8657	-0.7144
0.0715	0.8807	-0.7762
0.0532	0.8963	-0.8473

TABLE II (Cont.)

X/R_E	R/R_E	$\tan\theta$
0.0386	0.9098	-0.9318
0.0265	0.9224	-1.0315
0.0158	0.9346	-1.1513
0.0080	0.9449	-1.3021
0.0018	0.9541	-1.4932
-0.0034	0.9633	-1.7348
-0.0071	0.9709	-2.0670
-0.0095	0.9771	-2.5409

Sonic line

$\phi = -2.9357$ rad $\qquad R_T/R_E = 0.9962$

DISCUSSION

The Prandtl-Meyer flow theory was applied to the lip of the nozzle so that a series of two-dimensional expansion waves originating at the nozzle lip can be determined. This method is generally used to treat the flow expanding around a corner, and was shown (Reference 8) to be in good agreement with experimental results.

It has previously been determined (Reference 7) that the numerical determination of flow field by the characteristics methods does not produce accurate results in the regions where Mach number is less than 1.15; therefore it was decided to establish the flow field initial Mach number equal to or greater than 1.15. In some of the examples presented in this report, the initial Mach numbers equal 1.3.

In solving M^* from Equations (38) and (39) to determine the flow properties along the control surface, there are three roots that mathematically satisfy these two equations, but the root with the largest value is the only one that physically satisfies these equations. The other two roots would cause discontinuity in flow properties along the control surface.

A comparison of the results of the present program with the one in Reference 6 was made. The input conditions were the same, and the difference in nozzle contour is shown in Figure 9. If the ideal plug is chopped off so that the nozzle length is the same as the optimum thrust plug, the vacuum thrust coefficient would be equal to 1.576 while the optimum

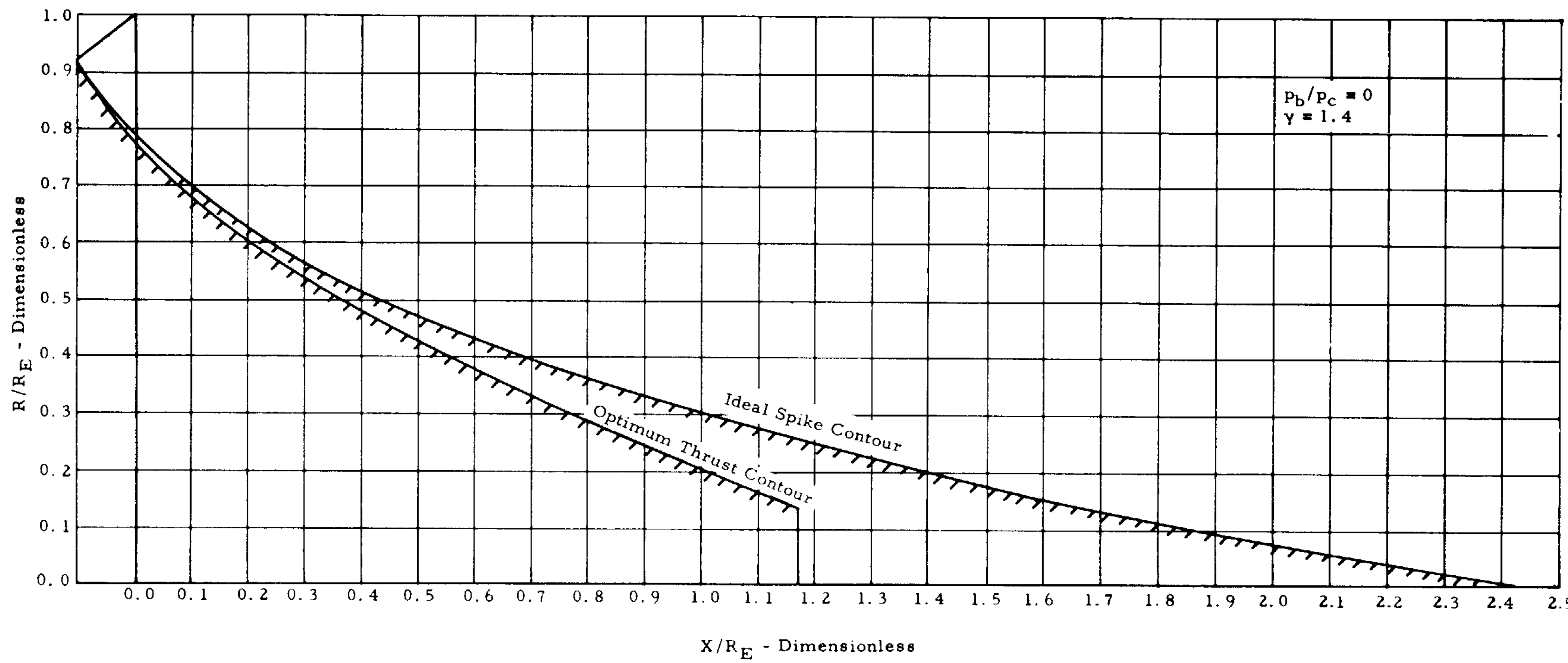


Figure 9

Comparison of Ideal and Truncated Spike Contours

thrust nozzle yields a thrust coefficient 1.579. To design a plug nozzle by using the present program, the nozzle yields 0.19% higher thrust coefficient and has less surface area than the ideal plug nozzle.

The base pressure at the end of the plug nozzle is one of the input conditions for the design program. In general, the base pressure depends upon the geometry of plug and flow conditions at D. If the data of base pressure are available, the designer may input his data to obtain more accurate plug contours. In most of the examples presented in this report, the base pressure was assumed to be zero. The effect of the base pressure on plug contour is shown in Figure 10.

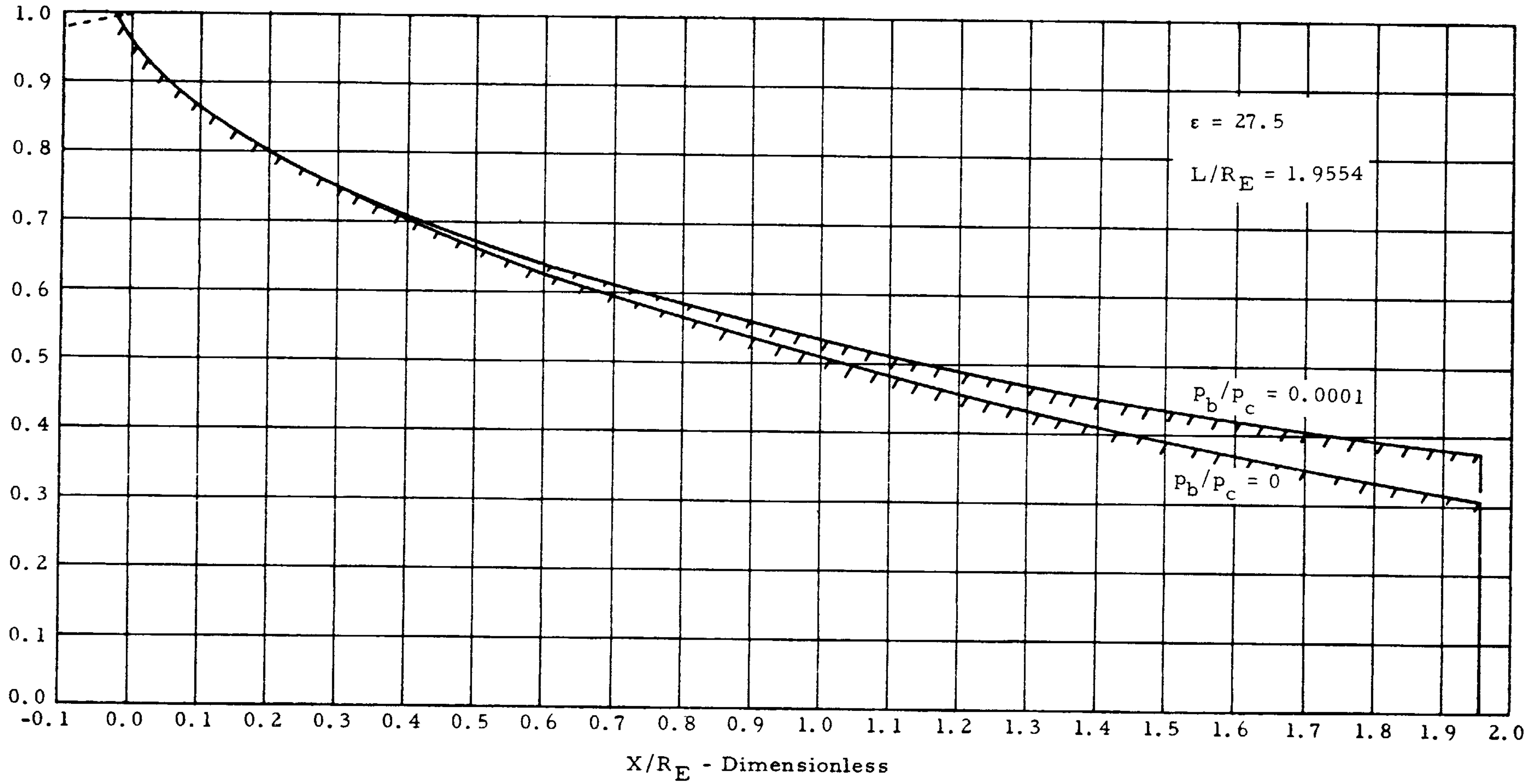


Figure 10

Effect of Base Pressure in Plug Contours

REFERENCES

1. G.V.R. Rao, "Spike Nozzle Contour for Optimum Thrust", Ballistic Missile and Space Technology, Vol. 2, C. W. Morrow (ed.) Pergamon Press, New York, 1961

2. G.V.R. Rao, "Exhaust Nozzle Contour for Optimum Thurst", ARS Semi-Annual Meeting, San Francisco, California, June, 1957

3. L. E. Cole, "A Simplified General Method for A Solution of the Characteristic Equation for Axially Symmetric Rocket Nozzle", MPT-P&VE-P-62-5 NASA

4. A. H. Shapiro, "The Dynamics and Thermodynamics of Compressible Fluid Flow", The Ronald Press Company, New York, Vol. I and II

5. J. S. Isenberg, "The Method of Characteristics in Compressible Flow", Part I, No. F-TR-1173A-ND, Graduate Division of Applied Mathematics, Brown University

6. C. C. Lee, "FORTRAN Programs for Plug Nozzle Design", Technical Note R-41, Brown Engineering Company, Inc., March, 1963

7. R. B. Dillaway, "A Philosophy for Improving Rocket Nozzle Design", presented at the ARS 11th Annual Meeting, New York, Nov. 26-29, 1956

8. E. S. Love, "Experimental and Theoretical Studies of Axisymmetric Free Jets", NASA Technical Report R-6, Langley Research Center, Langley Field, Virginia

APPENDIX

FORTRAN SYMBOLS

In the program of data generation

XME	M_E	Mach number at the lip of the shroud
DELME	ΔM_E	increment of M_E
THETE	θ_E	flow angle at E
DELTHE	$\Delta\theta_E$	increment of θ_E
GAM	γ	ratio of specific heats
AAA	$(\theta_E)_{max}$	limit value of θ_E
BBB	$(M_E)_{max}$	limit value of M_E
PBPC	$\frac{p_b}{p_c}$	base pressure ratio
RARAT	$\frac{\pi R_E^2}{A^*}$	expansion ratio
SUMLS	$\frac{L}{R_E}$	nozzle length in dimensionless form
CF	C_F	vacuum thrust coefficient

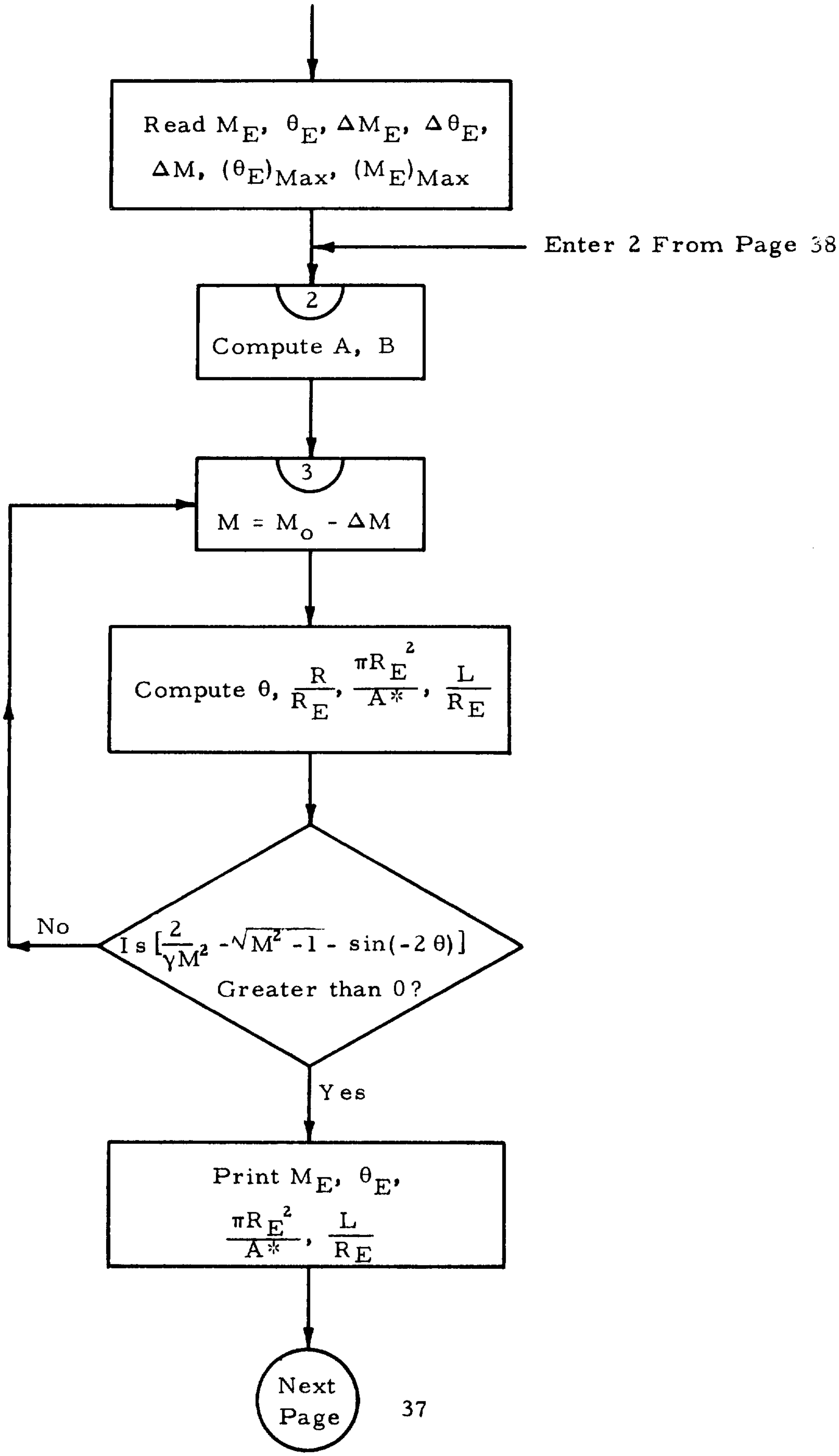
In the design program

XME	M_E	Mach number at the lip of the shroud
XMO	M_o	initial Mach number of the Prandtl Meyer expansion
XL	L	length of a nozzle
THE	θ_E	flow angle at E
RE	R_E	radius at E
GAM	γ	ratio of specific heats
N		number of right running characteristics

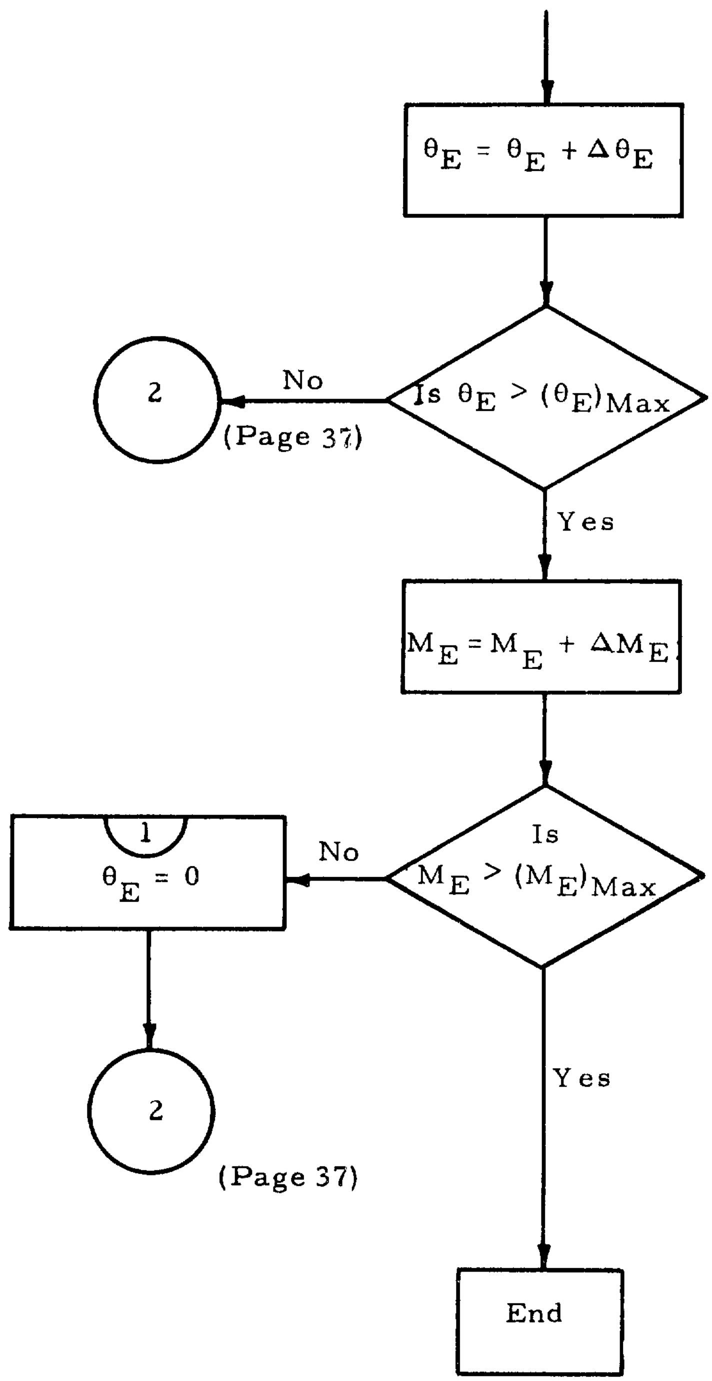
N1		number of left running characteristics
EPL	ε	expansion ratio
$\frac{PA}{PC}$	$\frac{p_a}{p_c}$	ambient pressure ratio
A1	x	X-coordinate
A2	R	R-coordinate
A3	M	Mach number
A4	M^*	dimensionless speed
A5	θ	flow angle
XA		X-coordinate of contour
YA		R-coordinate of contour
TERM	$\tan \theta$	inclination slope of contour
IPT1, IPT2		signifies the points in flow field
THESTR	θ^*	flow angle at the throat
PHISTR	ϕ^*	angle between sonic line and nozzle axis
RT	R_T	radius of throat

FORTRAN PROGRAM FOR INPUT DATA GENERATION

Flow Chart For Input Data Generation



Enter From Previous Page



LIST OF FORTRAN PROGRAM

DATA GENERATOR FOR SP-64

```
C                     DATA GENERATOR FOR SP-64
C
      ANGF%X¤#X/.17453293E-01
      COTF%X¤#COSF%X¤/SINF%X¤
      ARSINF%X¤#ATANF%X/%SQRTF %1.-X*X¤¤¤
      RADF%X¤#.17453293E-01*X
 20000READ  INPUT TAPE5,100,
     1          XME,DELME,THETE,DELTHE,DELM,GAM
  100 FORMAT%6E12.5¤
      READ INPUT TAPE 5,4,PBPC
    4 FORMAT%E12.5¤
      READ INPUT TAPE 5,1410,AAA,BBB,KODE
 1410 FORMAT%2F5.1,I2¤
      AAA#RADF%AAA¤
      DELTHE#RADF%DELTHE¤
      THETE#RADF%THETE¤
      GAM1#%GAM&1.¤/2.
      GAM2#%GAM-1.¤/2.
      GAM3#%1./%GAM-1.¤¤
      GAM4#%2./%GAM&1.¤¤
```

LIST OF FORTRAN PROGRAM

DATA GENERATOR FOR SP-64

```
      GAM5#GAM/%GAM-1.¤
    1 THETE#0.
    2 XMES#SQRTF%%GAM1*XME*XME¤/%1.&GAM2*XME*XME¤¤
      TNALPE#1./%SQRTF%XME *XME -1.¤¤
      A#XMES*%COSF%THETE¤&TNALPE*SINF%-THETE¤¤
      B#%XMES*SINF%-THETE¤¤**2*TNALPE*%1.&GAM2*XME *XME¤**%-GAM3¤
      XMO#XME
      THETO#THETE
      ALPE#ATANF%1./%SQRTF%XME**2-1.¤¤¤
      ALPO#ALPE
      RRATO#1.
      SUMRA#0.
      SUMLR#0.
      SUMF#0.
    3 XM#XMO-DELM
      TANALP#1./%SQRTF%XM*XM-1.¤¤
      ALP#ATANF%TANALP¤
      XMS#SQRTF%%GAM1*XM*XM¤/%1.&GAM2*XM*XM¤¤
      GAM6#%GAM-1.¤/%GAM&1.¤
      XM2#XMS*XMS
```

LIST OF FORTRAN PROGRAM

DATA GENERATOR FOR SP-64

```
XM1#XM2-1.
GT#%GAM4*XM2¤/XM1
AT1#%1.-GAM6*XM2¤/%XM2-1.¤
AT2#%-2.*A¤/XMS*SQRTF%AT1¤
AT3#%4.*A*A¤/XM2*AT1-4.*GT*%%A*A¤/XM2-1.¤
AT3#SQRTF%AT3¤
AT4#2.*GT
SINTHE#%AT2&AT3¤/AT4
THET#ARSINF%SINTHE¤
RRAT#B/%%XMS*SINF%-THET¤¤**2*TANALP*%1.&GAM2*XM*XM¤**%-GAM3¤¤
T1#%1./%%1.&GAM2*XMO*XMO¤*GAM4¤**GAM3¤
T2#SQRTF%%GAM1*XMO*XMO¤/%1.&GAM2*XMO*XMO¤¤*RRATO
T2#T2*SINF%ALPO¤/SINF%THETO-ALPO¤
T3#%1./%%1.&GAM2*XM*XM¤*GAM4¤**GAM3¤
T4#SQRTF%%GAM1*XM*XM¤/%1.&GAM2*XM*XM¤¤*RRAT
T4#T4*SINF%ALP¤/SINF%THET-ALP¤
T5#RRAT-RRATO
SUMRA#SUMRA&%T1*T2&T3*T4¤*T5
SUMLR#SUMLR&.5*%COTF%&THETO-ALPO¤&COTF%&THET-ALP¤¤*T5
F1#%1.&GAM2*XMO*XMO¤**%-GAM5¤
```

LIST OF FORTRAN PROGRAM

DATA GENERATOR FOR SP-64

```
  OF2#%1.&GAM*XMO*XMO*%%SINF%ALPO¤*COSF%-THETO¤¤/%SINF%-THETO&ALPO
  1¤¤¤¤*RRATO
   F3#%1.&GAM2*XMO*XMO¤**%-GAM5¤
  OF4#%1.&GAM*XM *XM *%%SINF%ALP ¤*COSF%-THET ¤¤/%SINF%-THET &ALP
  1¤¤¤¤*RRAT
   SUMF#SUMF&%F1*F2&F3*F4¤*%-T5¤
   RARAT#1./SUMRA
   CF#RARAT*SUMF
   TEST#%2./%GAM*XM*XM¤¤*SQRTF%XM*XM-1.¤-SINF%-2.*THET¤
   C#%1.0&GAM2*XM*XM¤**GAM5
   C1#%2./%GAM*XM*XM¤¤*SQRTF%XM*XM-1.¤
   C2#PBPC*C1*C
   TEST#TEST-C2
   IF%TEST¤10,10,20
20 XMO#XM
   THETO#THET
   ALPO#ALP
   RRATO#RRAT
   GO TO 3
10 THETE#ANGF%THETE ¤
```

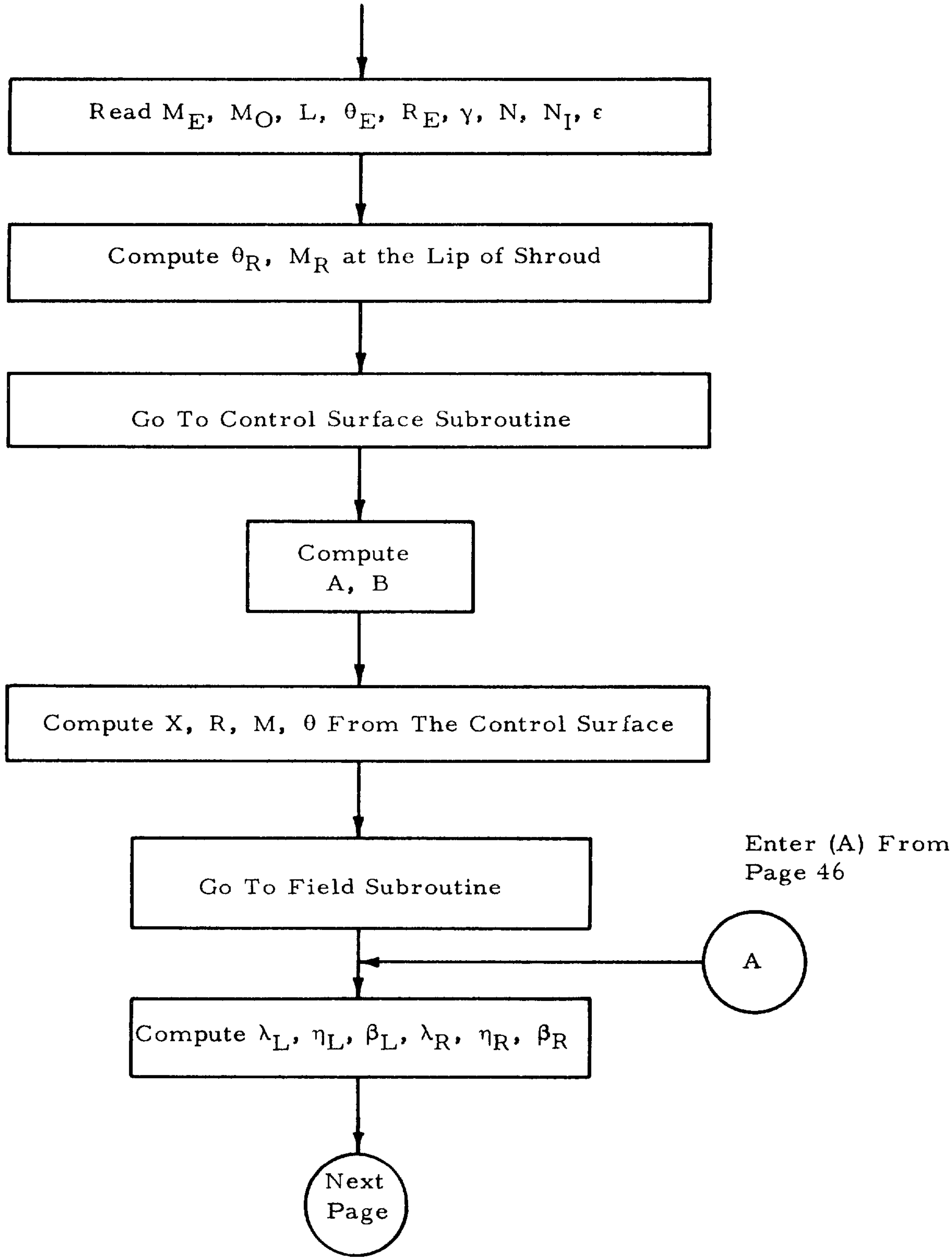
LIST OF FORTRAN PROGRAM

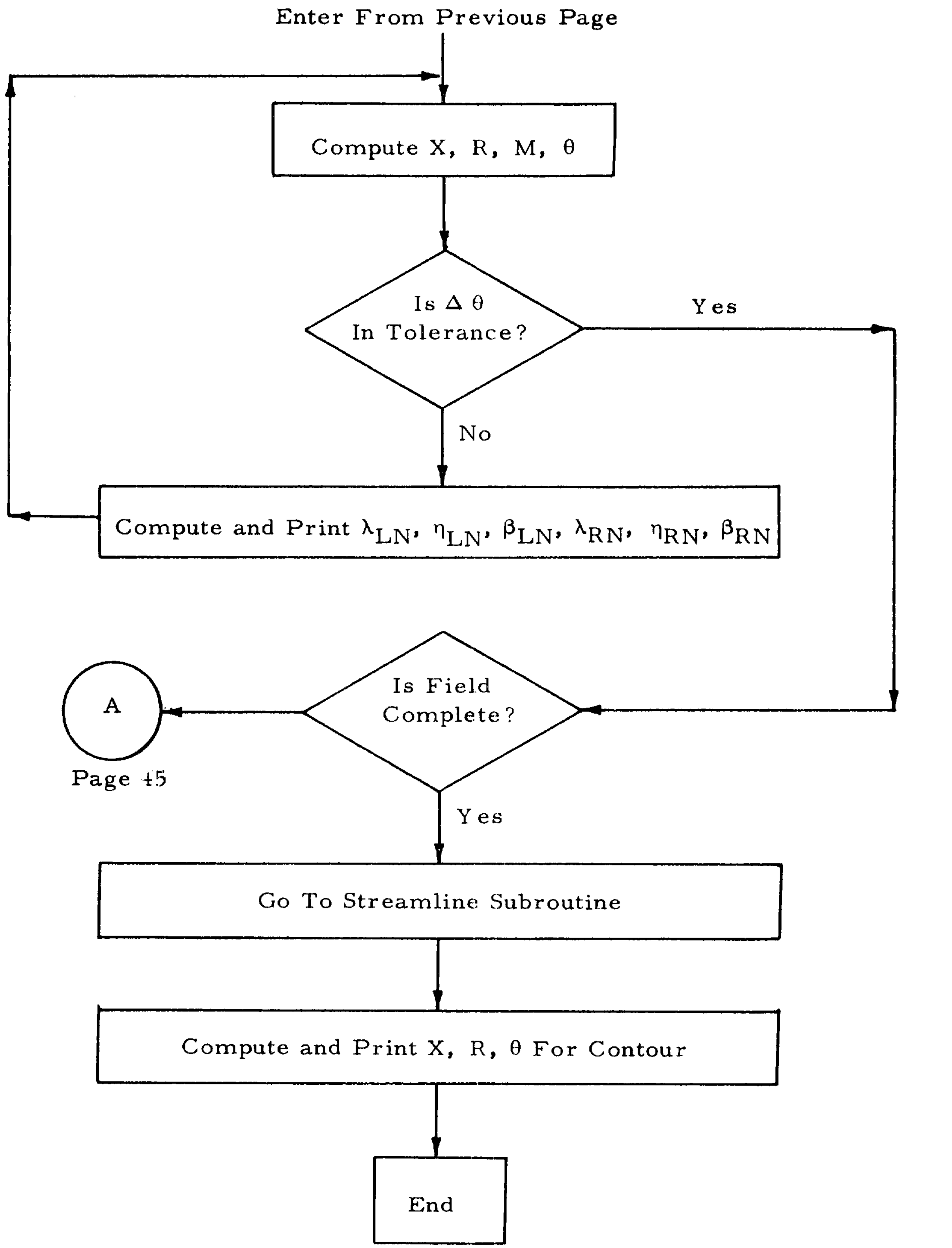
DATA GENERATOR FOR SP-64

```
    0 WRITE OUTPUT TAPE 6,200,
    1              XME,THETE,RARAT,SUMLR,CF
 2000FORMAT%1H0,5X,2HME,E17.8,5X,6HTHET E,E17.8,5X,4HR/A*, E17.8,5X,
    14HL/RE,E17.8,5X,2HCF,E17.8¤
      THETE#RADF%THETE ¤
      THETE#THETE-DELTHE
      IF%THETE&AAA¤11,11,2
   11 XME#XME&DELME
      IF%XME-BBB¤1,1,999
  999 CONTINUE
      IF%KODE¤2001,2000,2001
 2001 WRITE OUTPUT TAPE 6,222
  222 FORMAT%1H1////////////////////56X,22H***** END OF JOB *****/1H1¤
      CALL DUMP
      END
```

FORTRAN PROGRAM FOR OPTIMUM THRUST PLUG NOZZLE DESIGN

Flow Chart For Optimum Thrust Plug Nozzle Design




Enter From Previous Page
Compute X, R, M, θ
Is Δ θ In Tolerance?
Yes
No
Compute and Print λ_{LN}, η_{LN}, β_{LN}, λ_{RN}, η_{RN}, β_{RN}
A
Page 45
Is Field Complete?
Yes
Go To Streamline Subroutine
Compute and Print X, R, θ For Contour
End

LIST OF FORTRAN PROGRAM

FORTRAN PROGRAM FOR OPTIMUM THRUST PLUG NOZZLE DESIGN

```
      DIMENSION FX%1000¤,FY%1000¤,FTHE%1000¤
     0DIMENSION RX%35¤,RY%35¤,RTHE%35¤,RM%35¤,RMST%35¤,CX%35¤,CY%35¤,CM
     1%35¤,CMST%35¤,CTHE%35¤
      ASINF%X¤#ATANF%X/SQRTF%1.-X**2¤¤
      TANF%X¤#SINF%X¤/COSF%X¤
20010READ INPUT TAPE 5,100,
     1         XME,XMO,XL,THE,RE,GAM
 100 FORMAT%6E13.6¤
 101 FORMAT%3I2¤
      READ INPUT TAPE 5,101,N,N1,KODE
      READ INPUT TAPE5, 100,EPL
      WRITE OUTPUT TAPE 6,2020,XME,XMO,XL,THE,RE,GAM,N,N1,EPL
20200FORMAT%1H1,2HME,E15.6,3X,2HMO,E15.6,3X,2HXL,E15.6,3HTHE,E15.6,
     13X,2HRE,E15.6,3X,3HGAM,E15.6/1H0,1HN,I4,3X,2HN1,I4/1H0,3HEPL,E15.6
     2¤
      ALE#ASINF%1./XME¤
      C1#%1.&%GAM-1.¤*XME*XME/2.¤ **%-GAM/%GAM-1.¤¤
      C2#1.-0.5*SINF%-2.*THE¤*XME*XME*SINF%ALE¤/COSF%ALE¤
      PAPC#C1*C2
      WRITE OUTPUT TAPE 6,2003,PAPC
```

FORTRAN PROGRAM FOR OPTIMUM THRUST PLUG NOZZLE DESIGN

```
2003 FORMAT%1H0,5HPA/PC,E15.6¤
     NVAL#%N&2¤*N1
     JJ#N&1
     XJJ#JJ
     DELM#%XME-XMO¤/XJJ
     WRITE OUTPUT TAPE 6,103
 1030FORMAT%1H1,9X,1HX,14X,1HY,14X,1HM,13X,2HM*,12X,5HTHETA,5X,10HITERA
    1TIONS¤
     DO 1 I#1,JJ
     RX%I¤#0.0
     RY%I¤#RE
     XI#I
     RM%I¤#XME-XI*DELM
    ORTHE%I¤#THE-SQRTF%%GAM&1.¤/%GAM-1.¤¤*ATANF%SQRTF%%GAM-1.¤/%GAM&1.¤
    1*%XME**2-1.¤¤¤
     RTHE%I¤#RTHE%I¤&ATANF%SQRTF%XME**2-1.¤¤
    ORTHE%I¤#RTHE%I¤&SQRTF%%GAM&1.¤/%GAM-1.¤¤*ATANF%SQRTF%%GAM-1.¤/%GAM
    1&1.¤*%RM%I¤**2-1.¤¤¤
     RTHE%I¤#RTHE%I¤-ATANF%SQRTF%RM%I¤**2-1.¤¤
     RMST%I¤#SQRTF%%%GAM&1.¤/2.*RM%I¤**2¤/%1.&%GAM-1.¤/2.*RM%I¤**2¤¤
```

FORTRAN PROGRAM FOR OPTIMUM THRUST PLUG NOZZLE DESIGN

```
      1 CONTINUE
        ICTR#0
        CALL CONTRL%XME,THE,XL,N1,RE,GAM,CX,CY,CM,CMST,CTHE¤
        REWIND 8
        DO 2 I#1,N1
        WRITE OUTPUT TAPE 8,102,CX%I¤,CY%I¤,CM%I¤,CMST%I¤,CTHE%I¤,ICTR
    102 FORMAT%5E13.6,I3¤
        I40#I*%N&2¤
        FX%I40¤#CX%I¤
        FY%I40¤#CY%I¤
        FTHE%I40¤#CTHE%I¤
        AAA#CX%I¤
        BBB#CY%I¤
        CCC#CM%I¤
        DDD#CMST%I¤
        EEE#CTHE%I¤
        DO 3 J#1,JJ
        ICTR#0
       0CALL FIELD%RX%J¤,AAA,BBB,RY%J¤,CCC,RM%J¤,EEE,RTHE%J¤,DDD,RMST%J¤,
       1GAM,X4,Y4,XMS4,TH4,XM4,ICTR¤
```

FORTRAN PROGRAM FOR OPTIMUM THRUST PLUG NOZZLE DESIGN

```
      IF%ICTR-30¤250,2001,250
  250 WRITE OUTPUT TAPE 8,102,X4,Y4,XM4,XMS4,TH4,ICTR
      I41#I40-J
      FX%I41¤#X4
      FY%I41¤#Y4
      FTHE%I41¤#TH4
      AAA#X4
      BBB#Y4
      CCC#XM4
      DDD#XMS4
      EEE#TH4
    3 CONTINUE
      REWIND 8
      READ INPUT TAPE 8,102,A1,A2,A3,A4,A5,ICTR
      DO 4 J1#1,JJ
      READ INPUT TAPE 8,102,A1,A2,A3,A4,A5,ICTR
      RX%J1¤#A1
      RY%J1¤#A2
      RM%J1¤#A3
      RMST%J1¤#A4
```

FORTRAN PROGRAM FOR OPTIMUM THRUST PLUG NOZZLE DESIGN

```
      RTHE%J1¤#A5
    4 CONTINUE
      REWIND 8
      JKL#N&2
      DO 5 J2#1,JKL
      READ INPUT TAPE 8,102,A1,A2,A3,A4,A5,ICTR
      WRITE OUTPUT TAPE 6,104,A1,A2,A3,A4,A5,ICTR
  104 FORMAT%1H ,5E15.6,9X,I3¤
    5 CONTINUE
      REWIND 8
    2 CONTINUE
      CALL STREAM%FX,FY,FTHE,NVAL,N ,THE,GAM,XME,EPL,RE¤
      IF%KODE¤2002,2001,2002
 2002 CALL DUMP
      END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE FIELD

```
*       LIST 8
       OSUBROUTINE FIELD %XR,XL,RL,RR,XML,XMR,OL,OR,XMLS,XMRS,GAM,XNN,RNN,
       1XMNSN,ONN,XMN,ICTR¤
        RADF%X¤#.17453293E-01*X
        TANF%X¤#SINF%X¤/COSF%X¤
        COTF%X¤#COSF%X¤/SINF%X¤
        ANGF%X¤#X/.17453293E-01
        ASINF%X¤#ATANF%X/SQRTF%1.-X**2¤¤
        AL#ASINF%1./XML¤
        ALL#TANF%OL&AL¤
        AR#ASINF%1./XMR¤
        ALRP#TANF%OR-AR¤
        BL#%SINF%OL¤*SINF%AL¤¤/%RL*COSF%OL&AL¤¤
        BRP#%SINF%OR¤*SINF%AR¤¤/%RR*SINF%OR-AR¤¤
        XNUL#COTF%AL¤/XMLS
        XNUR#COTF%AR¤/XMRS
C
C                       INITIAL CALCULATION OF OUTPUT
C
        XN#%%ALRP*XR-ALL*XL¤&%RL-RR¤¤/%ALRP-ALL¤
```

LIST OF FORTRAN PROGRAM

SUBROUTINE FIELD

```
      RN#RL-ALL*%XL-XN¤
     0XMNS#%OR-OL&XNUL*XMLS&XNUR*XMRS-BRP*%RR-RN¤-BL*%XL-XN¤¤/%XNUL
     1&XNUR¤
      ON#OL-XNUL*%XMLS-XMNS¤&BL*%XL-XN¤
      ON#ANGF%ON¤
      ON#RADF%ON¤
C
C
C
      GAM1#2./%GAM&1.¤
      GAM2#%GAM-1.¤/%GAM&1.¤
      DO 100 I#1,30
      ICTR#ICTR&1
      XMN#SQRTF%%GAM1*XMNS*XMNS¤/%1.-GAM2*XMNS*XMNS¤¤
      AN#ASINF%1./XMN¤
      ALN#TANF%ON&AN¤
      ALNP#TANF%ON-AN¤
      XNUN#COTF%AN¤/XMNS
      BN#%SINF%ON¤*SINF%AN¤¤/%RN*COSF%ON&AN¤¤
      BNP#%SINF%ON¤*SINF%AN¤¤/%RN*SINF%ON-AN¤¤
```

SUBROUTINE FIELD

```
C
C
C
      ALLN#%ALL&ALN¤/2.
      ALRNP#%ALRP&ALNP¤/2.
      XNULN#%XNUL&XNUN¤/2.
      BLN#%BL&BN¤/2.
      BRNP#%BRP&BNP¤/2.
      XNN#%%ALRNP*XR-ALLN*XL¤&%RL-RR¤¤/%ALRNP-ALLN¤
      XNURN#%XNUR&XNUN¤/2.
      RNN#RL-ALLN*%XL-XNN¤
     0XMNSN#%OR-OL&XNULN*XMLS&XNURN*XMRS-BRNP*%RR-RNN¤-BLN*%XL-XNN¤¤
     1/%XNULN&XNURN¤
      ONN#OL-XNULN*%XMLS-XMNSN¤&BLN*%XL-XNN¤
      IF%ABSF%%%ONN-ON¤/ONN¤¤-.1E-03¤40,40,50
   50 ON#ONN
      RN#RNN
      XMNS#XMNSN
      XN#XNN
  100 CONTINUE
```

SUBROUTINE FIELD

```
      WRITE OUTPUT TAPE 6,200,ICTR
2000FORMAT%1H0,26HNO SOLUTION IN FIELD AFTER,I3,32H ITERATIONS PROCEED
     1 TO NEXT CASE¤
   40 RETURN
      END
```

SUBROUTINE CONTRL

```
*     LIST 8
      SUBROUTINE CONTRL%XME,THE,XL,N1,RE,GAM,X,R,XM,XMSN,TH¤
C
C                                   4 JUNE 1963
C                         MODIFICATION 1 ON 6 JUNE 1963
C
      DIMENSION X%35¤,R%35¤,XM%35¤,XMSN%35¤,TH%35¤,AL%35¤
      ARSINF%XX¤ # ATANF%XX/SQRTF%1.-XX*XX¤¤
      TANF%Y¤ # SINF%Y¤/COSF%Y¤
C     PRINT 501
C 501 FORMAT%1H1,23X,1HX,19X,1HR,18X,2HXM,15X,6HXMSTAR,11X,5HTHETA¤
      NPTS # 0
      DX # XL/FLOATF%N1¤
      XO # 0.
      XI # DX
      GAM1 # GAM &1.
      GAM2 # GAM -1.
      XME2 # XME*XME
      XMES # SQRTF%%GAM1*XME2/2.¤/%1. &%GAM2*XME2/2.¤¤¤
      ALE # ARSINF%1./XME¤
```

LIST OF FORTRAN PROGRAM

```
      SUBROUTINE CONTRL


      TH1#THE
      AL1#ALE
      RI2#RE
      RI # RE & XI*TANF%THE - ALE¤
      A # XMES*%COSF%THE¤ & TANF%ALE¤*SINF%-THE¤¤
      B # RE*%%XMES * SINF%-THE¤¤**2¤*TANF%ALE¤*%1./%1. & GAM2*XME2/2.¤*
     1*%1./GAM2¤¤
      DO 71 I # 1,30
  106 XMS#XMES
      KK # 0
       K # 0
      DEL#0.0001
C
C                 CALCULATION OF SINE OF THETA
C
   12 XMS2 # XMS*XMS
      XMS21 # XMS2 -1.
      T1 # 1. - GAM2*XMS2/GAM1
      TERM1 # 2. * A * SQRTF%ABSF%T1/XMS21¤¤/XMS
      T2 # 4.*A*A*%T1/XMS21¤/XMS2
```

SUBROUTINE CONTRL

```
      T3 # 4.*%%2./GAM1□*XMS2/XMS21□*%%A*A/XMS2□-1.□
      TERM2 # SQRTF%ABSF%T2 - T3□□
      TERM3 # 2.*%2.*XMS2/GAM1□/XMS21
      SINTH # %-TERM1 & TERM2□/TERM3
C
C                    CALCULATION OF DIFFERENCE
C
      D1 # %2.*XMS2/GAM1□/T1
      D2 # 1. &%GAM2/2.□*D1
C     DT1 # RI/D2**%1./GAM1□                        CHANGED BY MOD1
      DT1 # RI/D2**%1./GAM2 □
      DT2 # %%XMS*SINTH□**2□*SQRTF%ABSF%T1/XMS21□□
      DIFF # DT1*DT2 - B
      IF%ABSF%DIFF□-.1E-05□105,105,56
  105 RI1#RI
      TH2#ARSINF%SINTH□
      XM2#SQRTF%%2.*XMS2/GAM1□/%1.-%GAM2*XMS2/GAM1□□□
      AL2#ARSINF%1./XM2□
      RI#RI2&DX*%TANF%TH1-AL1□&TANF%TH2-AL2□□/2.
      IF%ABSF%RI1-RI□-.1E-03□55,55,106
```

LIST OF FORTRAN PROGRAM

SUBROUTINE CONTRL

```
   56 IF%K -1¤10,11,10
   10 K # 1
      DIFFO # DIFF
      XMS#XMS-DEL
      GO TO 12
   11 IF%DIFFO¤50,51,51
   51 IF%DIFF¤52,53,53
   50 IF %DIFF¤53,53,52
C
C                         NO SIGN CHANGE
C
   53 DIFFO # DIFF
      XMS#XMS-DEL
      GO TO 12
C
C                         SIGNS CHANGED
C
   52 XMS#XMS&DEL
      DEL # DEL * 0.1
      XMS#XMS-DEL
```

SUBROUTINE CONTRL

```
      GO TO 12
   55 TH%I¤ # ARSINF%SINTH¤
      XM%I¤ # SQRTF%%2.*XMS2/GAM1¤/%1.-%GAM2*XMS2/GAM1¤¤¤
      AL%I¤ # ARSINF%1./XM%I¤¤
      XMSN%I¤ # XMS
      X%I¤ # XI
      R%I¤ # RI
      TH1#TH%I¤
      AL1#AL%I¤
      RI2#R%I¤
      NPTS # NPTS &1
C     PRINT 500,X%I¤,R%I¤,XM%I¤,XMSN%I¤,TH%I¤,NPTS
C 500 FORMAT%1HK,10X,5E20.6,5X,6HNPTS #,I7¤
      XI # X%I ¤ & DX
      RI # R%I ¤ & DX*TANF%TH%I ¤-AL%I ¤¤
      IF%XL - XI¤70,71,71
   71 CONTINUE
C     PRINT 502,XI,XL
C 502 FORMAT%/////10X,21HXI IS NOT EQUAL TO XL,5X,4HXI #,E16.6,5X,4HXL
C    1,E16.6¤
```

SUBROUTINE CONTRL

```
70 N1#NPTS
   RETURN
   END
```

LIST OF FORTRAN PROGRAM

SUBROUTINE STREAM

```
*       LIST 8
        SUBROUTINE STREAM%FX,FY,FTHE,NVAL,NPTS,THEE,GAM,XME,E,RE¤
        DIMENSION FX%1000¤,FY%1000¤,FTHE%1000¤,IEPT%100¤
        ANGF%X¤#X/ .17453293E-01
        RADF%X¤#.17453293E-01*X
        TANF%X¤#SINF%X¤/COSF%X¤
        SIGN1#0.
        SIGN2#0.
        IUP#0
        LEFT#0
        CHANGE#0.
        SOL#0.
        IEPT%1¤#1
        NLINE#NVAL/%NPTS&2¤
C
C               STREAMLINE SP-64
C               C.C. LEE
C               11 JUNE 1963
C               BY M. DOUGHTY
C
```

LIST OF FORTRAN PROGRAM

SUBROUTINE STREAM

```
      GAM1#%GAM&1.¤/%GAM-1.¤
      GAM2#%GAM-1.¤/%GAM&1.¤
     OTHESTR#THEE-SQRTF%GAM1¤*ATANF%SQRTF%GAM2*%XME*XME-1.¤¤¤&
     1ATANF%SQRTF%XME*XME-1.¤¤
      PHISTR#THESTR-RADF%90.¤
      RT#SQRTF%RE*RE-%RE*RE¤/E*COSF%THESTR¤¤
      WRITE OUTPUTTAPE 6,510,THESTR,PHISTR,RT
  5100FORMAT%1H0,19H LOCATION OF THROAT/1H ,7HTHETA *,E15.6,5X,5HPHI *,
     1E15.6,5X,2HRT,E15.6¤
      L#NVAL
      IPT1#NVAL
      DO 1410LEPT#2,NLINE
      LLL#LEPT-1
      IEPT%LEPT¤#IEPT%LLL¤&%NPTS&2¤
 1410 CONTINUE
      WRITE OUTPUT TAPE 6,500
  5000FORMAT%1H1,10HSTREAMLINE/1H0,5X,2HXA,13X,2HYA,13X,5HTHETA,8X,3HPT1
     1,3X,3HPT2¤
  600 IF%CHANGE-1.¤691,1111,691
 1111 IF%SOL-1.¤1112,1113,1112
```

LIST OF FORTRAN PROGRAM

SUBROUTINE STREAM

```
1112 IPT1#LPT1
     IPT2#LPT2
     CHANGE#0.
1113 SOL#0.
     CHANGE#0.
 691 IPT2#IPT1-1
     IPT1#IPT1-%NPTS&2¤
     TEST#0.
     SIGN1#0.
     SIGN2#0.
     IF%FX%IPT1¤-FX%L¤¤2000,777,777
2000 IF%FX%IPT2¤-FX%L¤¤1,777,777
   1 DELXA#%FX%IPT1¤-FX%IPT2¤¤/10.
     XA#FX%IPT2¤
11050TERM # %FY%L    ¤- FY %IPT1¤&%FX%IPT1¤-XA¤*%%FY%IPT1¤-FY%IPT2¤¤
    1/%FX%IPT1¤-FX%IPT2¤¤¤¤/%FX%L¤-XA¤
    OFUNCT#TERM                              -TANF%%%FX%IPT1¤-XA¤/%FX%IPT1¤
    1-FX%IPT2¤¤*FTHE%IPT2¤&%%XA-FX%IPT2¤¤/%FX%IPT1¤-FX%IPT2¤¤¤
    2*FTHE%IPT1¤¤¤
     WRITE OUTPUT TAPE6,602,FUNCT,IPT1,IPT2
```

SUBROUTINE STREAM

```
  602 FORMAT%1H ,E13.6,I6,I8¤
      WRITE OUTPUT TAPE6,1620,FX%IPT1¤,FX%IPT2¤,XA
 1620 FORMAT%1H ,5E13.6¤
      IF%ABSF%FUNCT¤-1.0E-04¤300,300,106
  106 IF%TEST¤1067,1066,1067
 1066 IF%FUNCT¤100,100,101
  100 SIGN1#1.
      IUP#1
      GO TO 102
  101 SIGN2#1.
      IUP#0
  102 IF%SIGN1-SIGN2¤104,103,104
 1067 IF%FUNCT¤1000,1000,1010
 1000 SIGN1#1.
      LEFT#1
      GO TO 102
 1010 SIGN2#1.
      LEFT#0
      GO TO 102
  103 XA#XA-DELXA
```

LIST OF FORTRAN PROGRAM

SUBROUTINE STREAM

```
      DELXA#DELXA*.1
      SIGN1#0.
      SIGN2#0.
      IF%DELXA¤4330,4330,4331
 4330 IF%FX%IPT1¤-XA¤105,105,200
 4331 IF%FX%IPT1¤-XA¤200,105,105
  105 CONTINUE
      GO TO 1105
  104 XA#XA&DELXA
      IF%DELXA¤4332,4332,4333
 4332 IF%FX%IPT1¤-XA¤105,105,200
 4333 IF%FX%IPT1¤-XA¤200,105,105
  200 IF%TEST¤775,777,775
  777 LPT1#IPT1
      LPT2#IPT2
      CHANGE#1.
      TEST#1.
      IPT1#IPT2
      IPT2#IPT1&%NPTS&1¤
      SIGN1#0.
```

SUBROUTINE STREAM

```
      SIGN2#0.
      GO TO 1
  775 IF%IUP-LEFT¤900,901,900
  900 XA#FX%LPT2¤
      YA#FY%LPT2¤
      IUP#0
      LEFT#0
      GO TO 666
  901 WRITE OUTPUT TAPE6,780
  780 FORMAT%1H1/////// 55X,       32HTRIED UP AND TO LEFT NO SOLUTION¤
      RETURN
  300 IF%TEST¤301,302,301
  301 SOL#1.
  3020YA#FY%IPT1¤-%FX%IPT1¤-XA¤*%%FY%IPT1¤-FY%IPT2¤¤/%FX%IPT1¤-FX%IPT2¤¤
     1¤
  666 WRITE OUTPUT TAPE 6,501,XA,YA,TERM,IPT1,IPT2
  501 FORMAT%1H ,3E15.6,2I6¤
      FX%L¤#XA
      FY%L¤#YA
      DO 1401 LLL#1,NLINE
```

SUBROUTINE STREAM

```
      IF%IEPT%LLL¤-IPT2¤1401,99,1401
 1401 CONTINUE
      GO TO 600
   99 PIE#3.1415926
      RETURN
      END
```